When Symmetry Does the Unexpected

Sal Pace

MIT

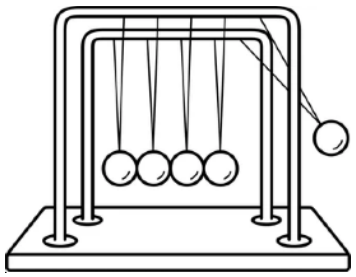
UMN Physics Colloquium

The Power of Symmetry

Symmetry is a foundation of modern physics

The Power of Symmetry

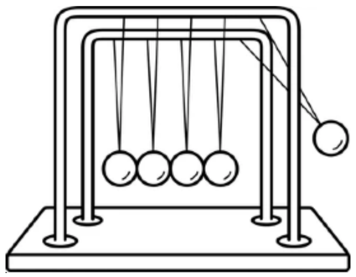
Symmetry is a foundation of modern physics



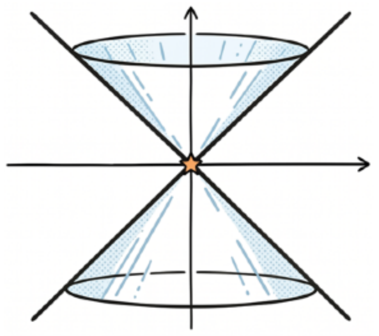
Classical mechanics: Conservation laws: Energy, momentum, angular momentum

The Power of Symmetry

Symmetry is a foundation of modern physics



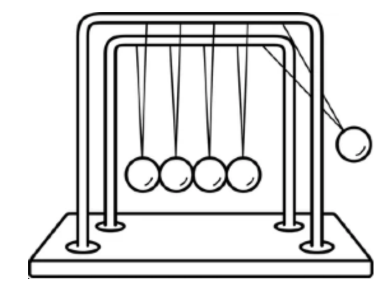
Classical mechanics: Conservation laws: Energy, momentum, angular momentum



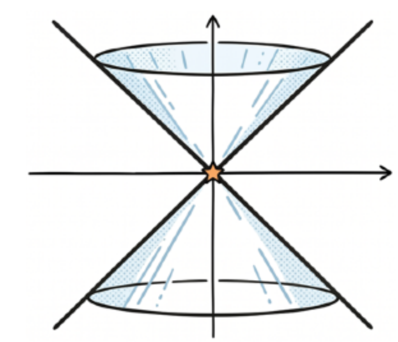
Relativity: Lorentz invariance of spacetime

The Power of Symmetry

Symmetry is a foundation of modern physics

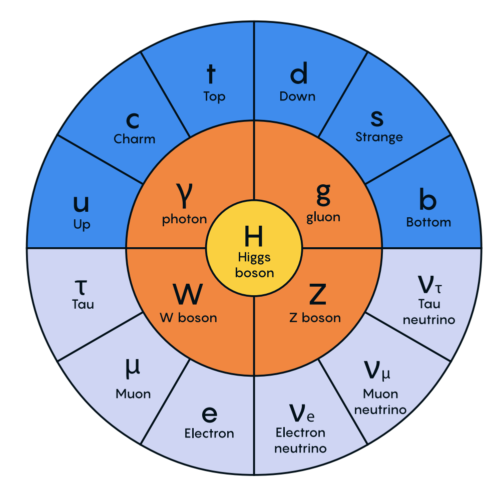


Classical mechanics: Conservation laws: Energy, momentum, angular momentum



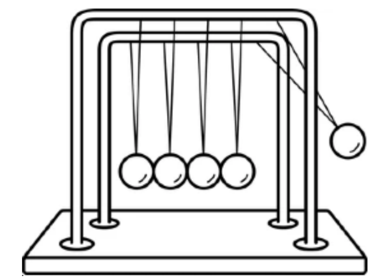
Relativity: Lorentz invariance of spacetime

Particle physics: Underlying principle of the Standard Model



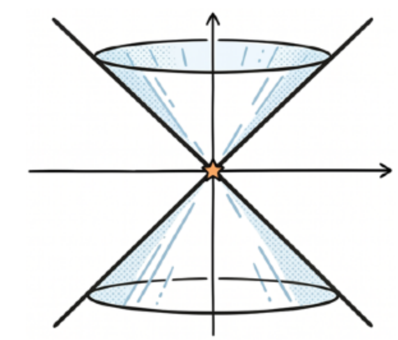
The Power of Symmetry

Symmetry is a foundation of modern physics

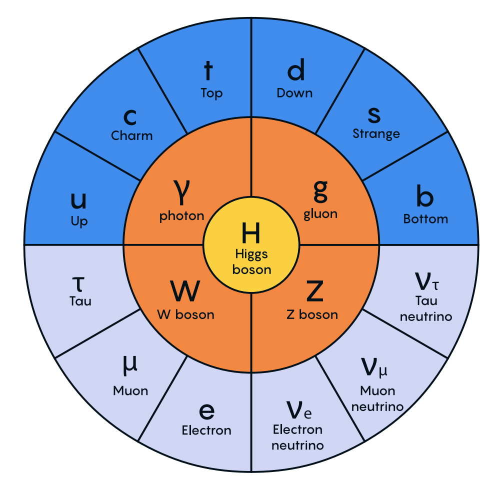


Classical mechanics: Conservation laws: Energy, momentum, angular momentum

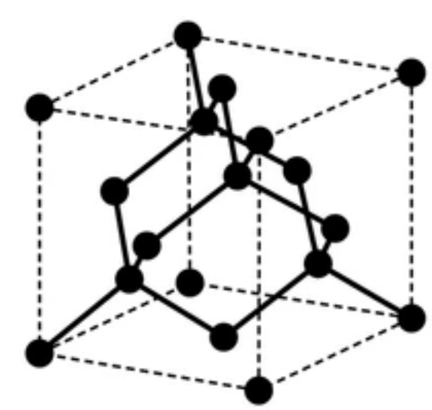
Relativity: Lorentz invariance of spacetime



Particle physics: Underlying principle of the Standard Model



Condensed matter: Phases of matter: Magnets, crystals, superfluids



Two Sides of Symmetry

Symmetry as a tool

Symmetry as a principle

Two Sides of Symmetry

Symmetry as a tool

Helps us understand a system

- Simplifies calculations
- Explains degeneracies and selection rules
- Gives conservation laws

Symmetry as a principle

Two Sides of Symmetry

Symmetry as a tool

Helps us understand a system

- Simplifies calculations
- Explains degeneracies and selection rules
- Gives conservation laws

Symmetry as a principle

Helps us decide what a system can be

- Determines allowed theoretical models
- Organizes phases of matter and universality classes

Two Sides of Symmetry

Symmetry as a tool

Helps us understand a system

- Simplifies calculations
- Explains degeneracies and selection rules
- Gives conservation laws

Symmetry as a principle

Helps us decide what a system can be

- Determines allowed theoretical models
- Organizes phases of matter and universality classes

Physics

Two Sides of Symmetry

Symmetry as a tool

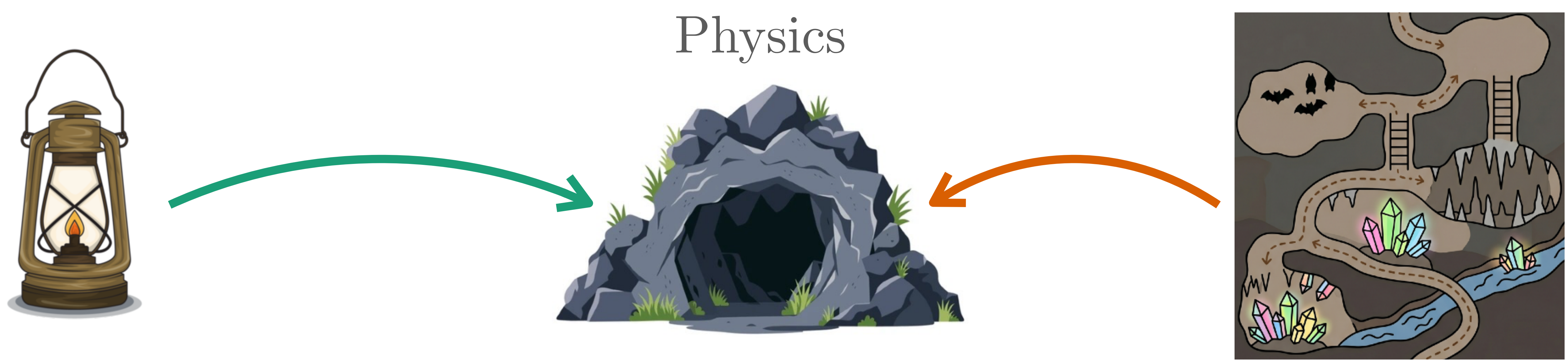
Helps us understand a system

- Simplifies calculations
- Explains degeneracies and selection rules
- Gives conservation laws

Symmetry as a principle

Helps us decide what a system can be

- Determines allowed theoretical models
- Organizes phases of matter and universality classes



Symmetry in Classical Physics

Symmetry in classical physics transforms classical states

Symmetry in Classical Physics

Symmetry in classical physics transforms classical states

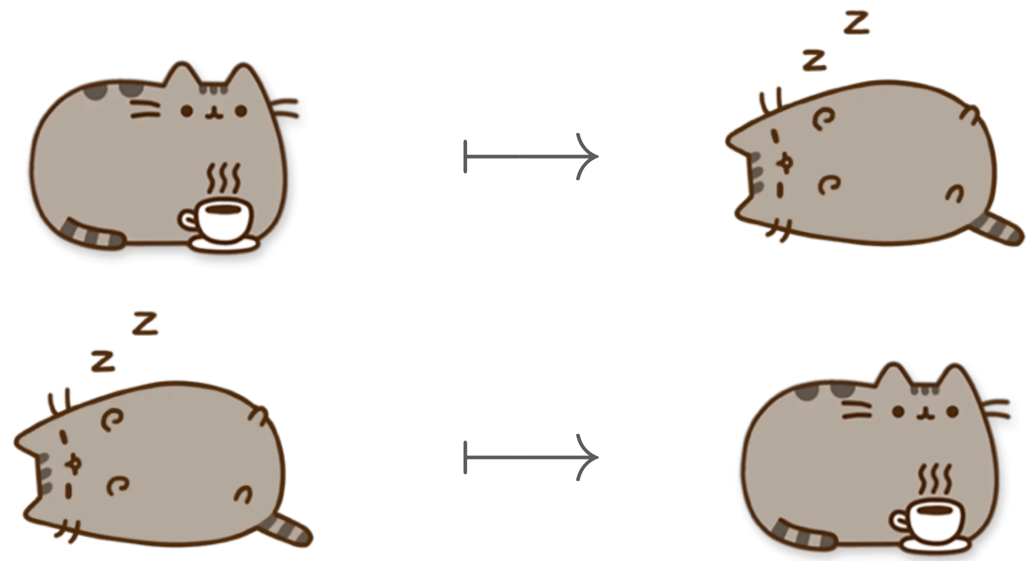
Symmetry in Classical Physics

Symmetry in classical physics transforms classical states

$\longmapsto$

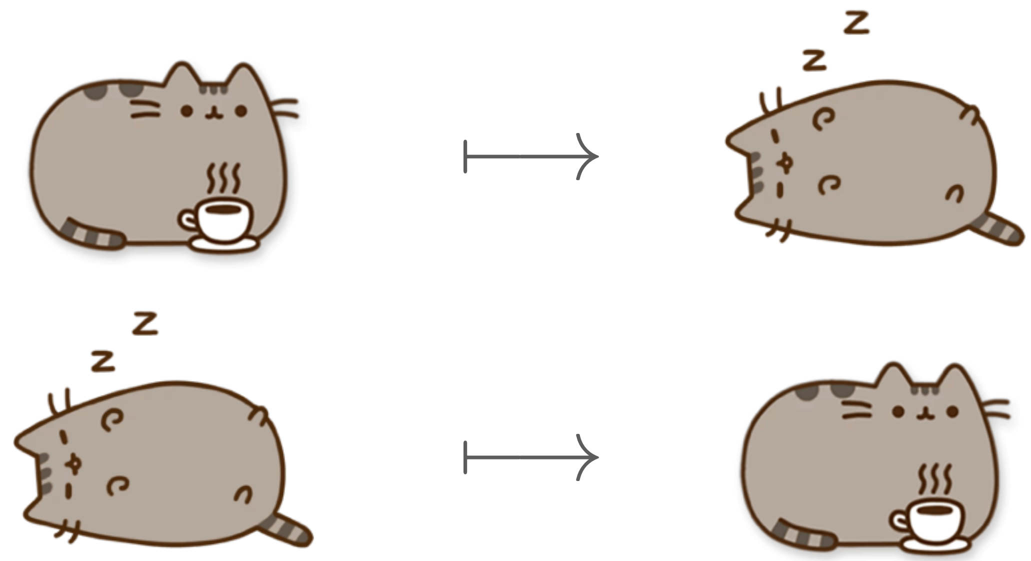
Symmetry in Classical Physics

Symmetry in classical physics transforms classical states



Symmetry in Classical Physics

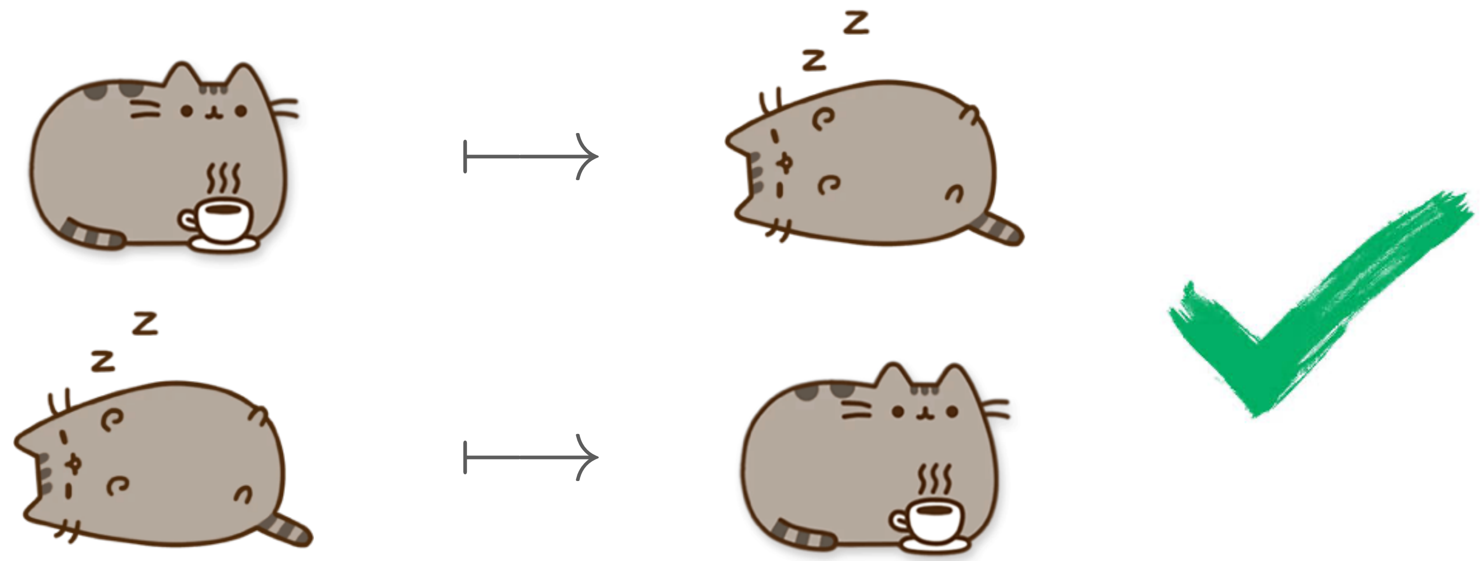
Symmetry in classical physics transforms classical states



- A symmetry commutes with classical time evolution

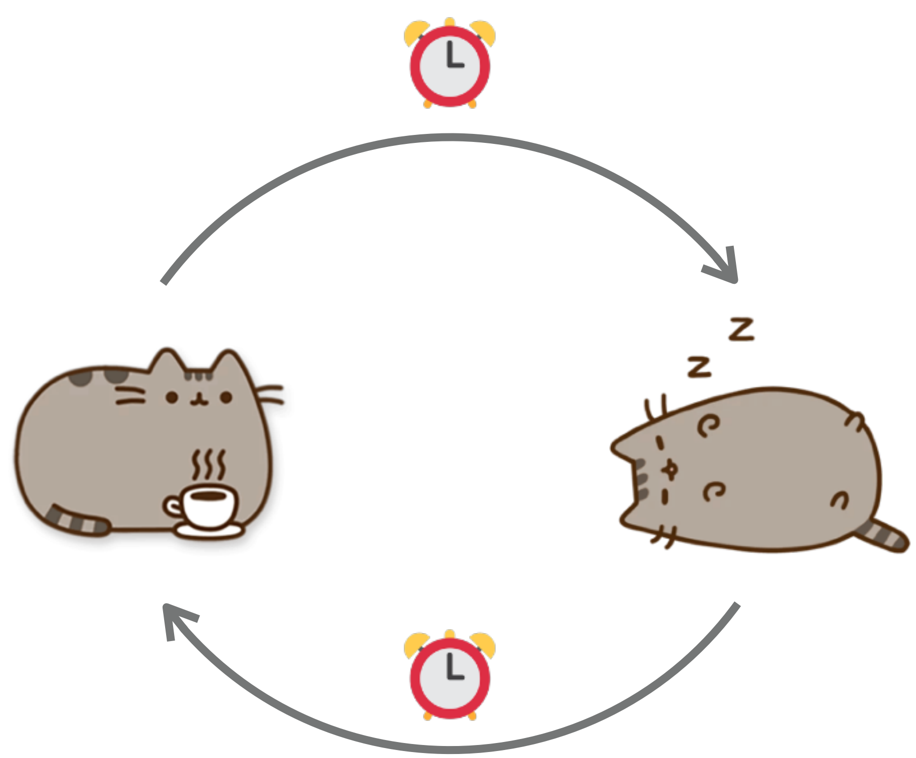
Symmetry in Classical Physics

Symmetry in classical physics transforms classical states



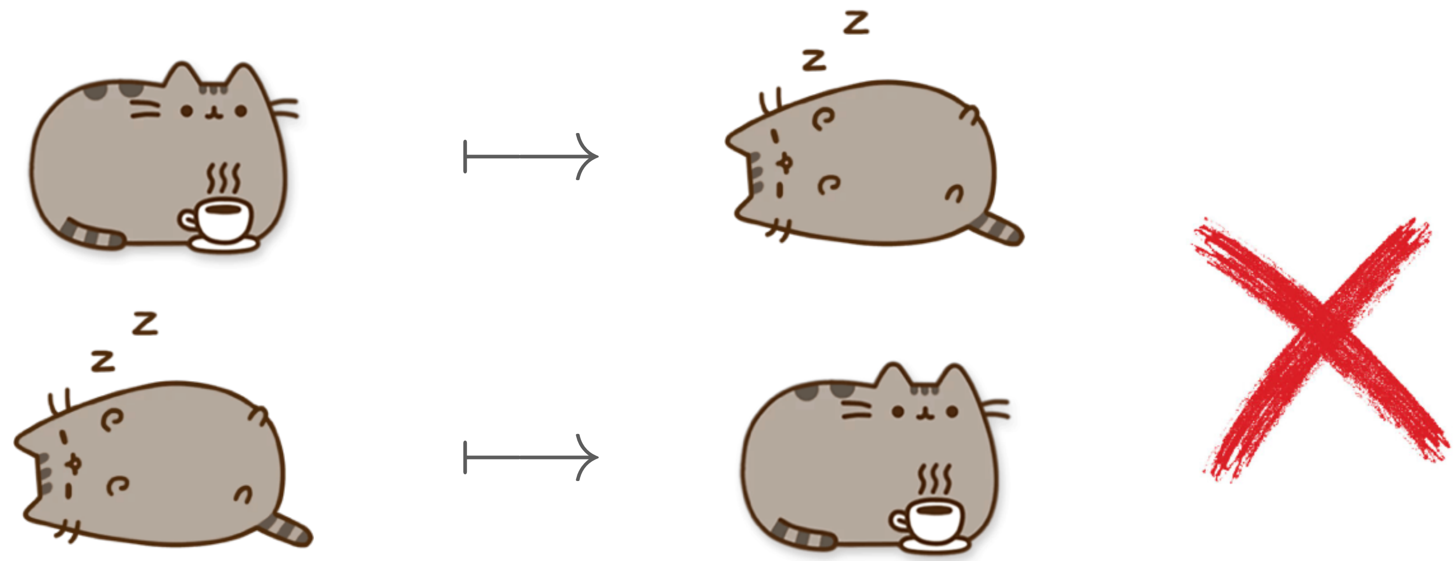
- A symmetry commutes with classical time evolution

Example:



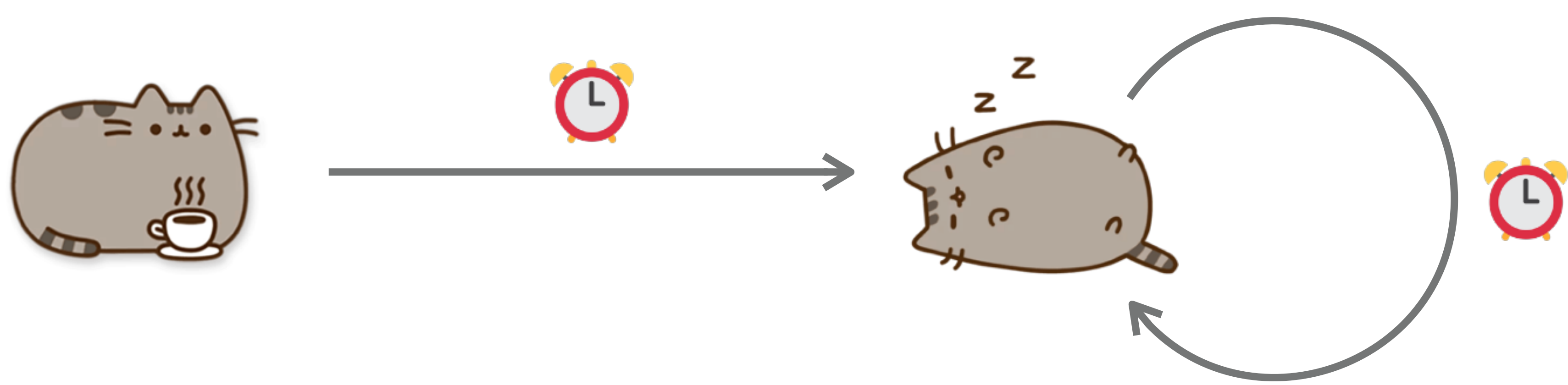
Symmetry in Classical Physics

Symmetry in classical physics transforms classical states



- A symmetry commutes with classical time evolution

Example:

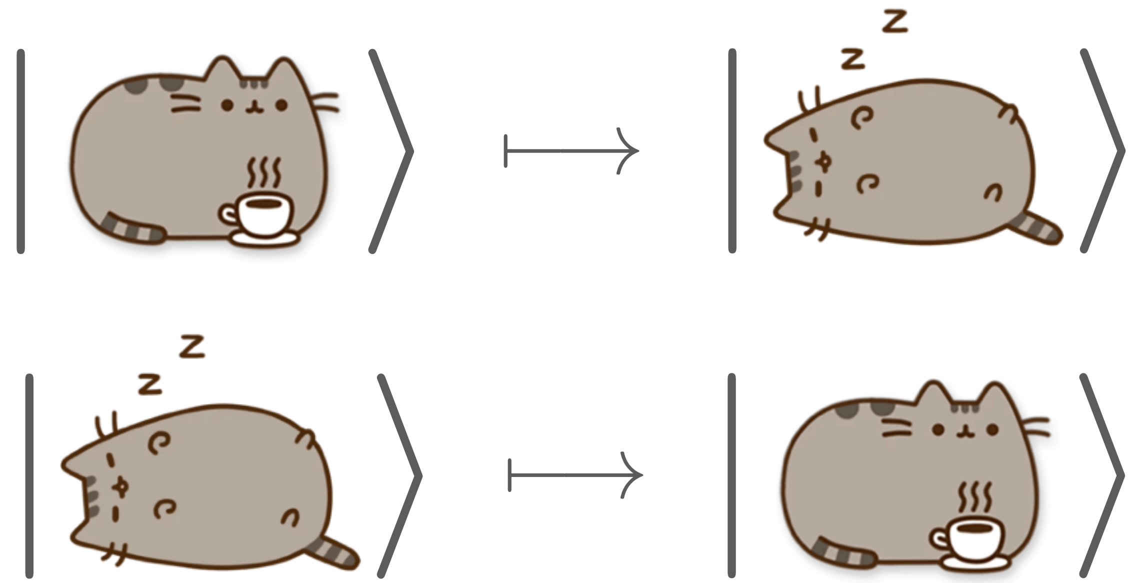


Symmetry in Quantum Physics

Symmetry in quantum physics transforms quantum states

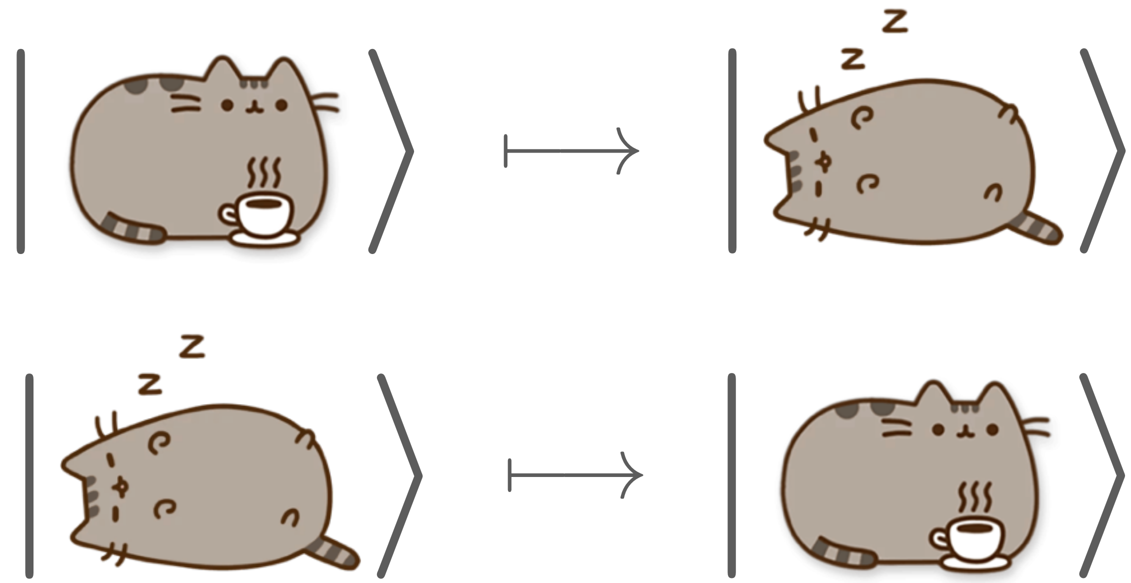
Symmetry in Quantum Physics

Symmetry in quantum physics transforms quantum states



Symmetry in Quantum Physics

Symmetry in quantum physics transforms quantum states

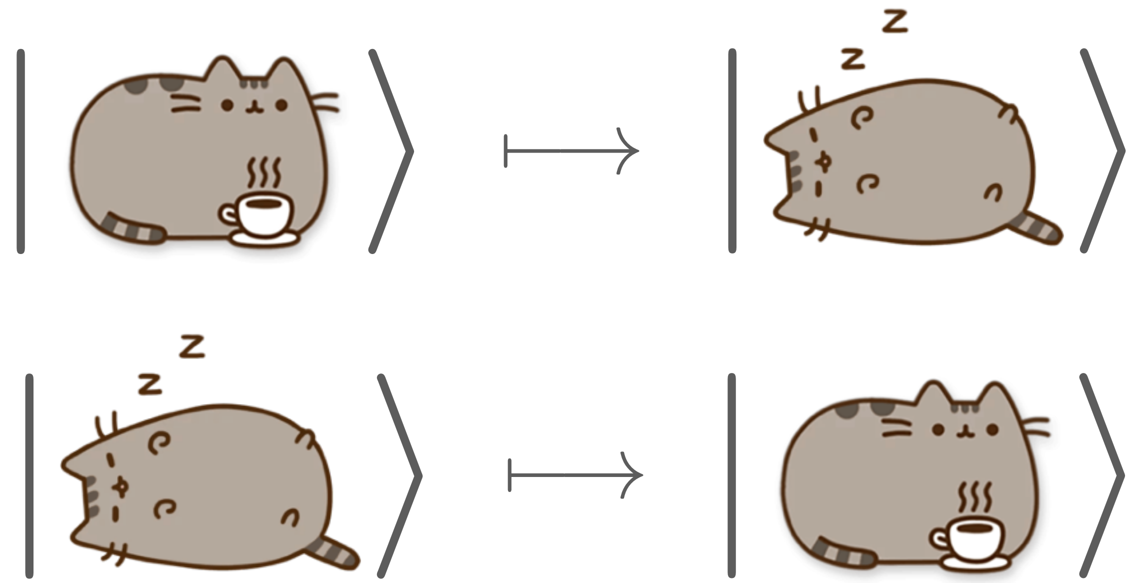


- A symmetry commutes with quantum time evolution

  (symmetry operator commutes with the Hamiltonian)

Symmetry in Quantum Physics

Symmetry in quantum physics transforms quantum states



- A symmetry commutes with quantum time evolution

  (symmetry operator commutes with the Hamiltonian)

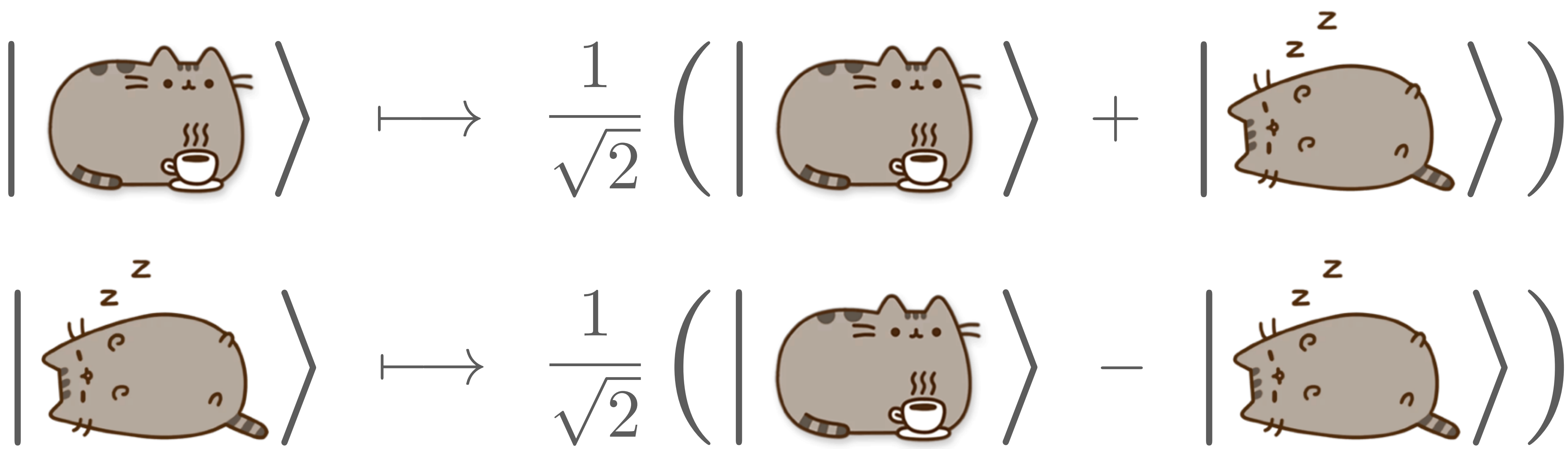
Two new quantum properties:

1. Superposition
2. Relative phases

Symmetry in Quantum Physics

Symmetry in quantum physics transforms quantum states

Example:



- A

(symmetry operator commutes with the Hamiltonian)

Two new quantum properties:

1. Superposition
2. Relative phases

Symmetry in Quantum Physics

Symmetry in quantum physics transforms quantum states

Symmetry is more interesting in quantum physics!

- A symmetry commutes with quantum time evolution (symmetry operator commutes with the Hamiltonian)

Two new quantum properties:

1. Superposition
2. Relative phases

Symmetry of a Spin-1/2

Consider a quantum spin-1/2: $|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$

Symmetry of a Spin-1/2

Consider a quantum spin-1/2: $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

- Pauli matrices:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X|\uparrow\rangle = |\downarrow\rangle, \quad X|\downarrow\rangle = |\uparrow\rangle \qquad Z|\uparrow\rangle = |\uparrow\rangle, \quad Z|\downarrow\rangle = -|\downarrow\rangle$$

Symmetry of a Spin-1/2

Consider a quantum spin-1/2: $|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$

- Pauli matrices:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X |\uparrow\rangle = |\downarrow\rangle , \quad X |\downarrow\rangle = |\uparrow\rangle \qquad Z |\uparrow\rangle = |\uparrow\rangle , \quad Z |\downarrow\rangle = - |\downarrow\rangle$$

Enforce X and Z as symmetry operators

Symmetry of a Spin-1/2

Consider a quantum spin-1/2: $|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$

- Pauli matrices:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X |\uparrow\rangle = |\downarrow\rangle, \quad X |\downarrow\rangle = |\uparrow\rangle \qquad Z |\uparrow\rangle = |\uparrow\rangle, \quad Z |\downarrow\rangle = - |\downarrow\rangle$$

Enforce X and Z as symmetry operators

$$X^2 = Z^2 = \mathbf{1} \qquad XZ = -ZX$$

- Mathematically: Projective $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

Symmetry of a Spin-1/2

Consider a quantum spin-1/2: $|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$

- Pauli matrices:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X\,|\uparrow\rangle = |\downarrow\rangle\,, \quad X\,|\downarrow\rangle = |\uparrow\rangle \qquad Z\,|\uparrow\rangle = |\uparrow\rangle\,, \quad Z\,|\downarrow\rangle = -\,|\downarrow\rangle$$

Enforce X and Z as symmetry operators

$$X^2 = Z^2 = \mathbf{1} \qquad XZ = -ZX$$

- Mathematically: Projective $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

Projective symmetry is intrinsically quantum!

Symmetry of a Spin-1/2

Consider a quantum spin-1/2: $|\psi\rangle = \alpha\,|{\uparrow}\rangle + \beta\,|{\downarrow}\rangle$

- Pauli matrices:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X\,|{\uparrow}\rangle = |{\downarrow}\rangle\,, \quad X\,|{\downarrow}\rangle = |{\uparrow}\rangle \qquad Z\,|{\uparrow}\rangle = |{\uparrow}\rangle\,, \quad Z\,|{\downarrow}\rangle = -\,|{\downarrow}\rangle$$

Enforce X and Z as symmetry operators

$$X^2 = Z^2 = \mathbf{1} \qquad XZ = -ZX$$

- Mathematically: Projective $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

Projective symmetry is intrinsically quantum!

- Consequence: Every symmetric Hamiltonian has degeneracies

Symmetry of a Spin-1/2

Consider a quantum spin-1/2: $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

Why?

Within any 1D eigenspace, X and Z must act as c-numbers.

But c-numbers commute $\Longrightarrow$ There are no 1D eigenspaces

Enforce X and Z as symmetry operators

$$X^2 = Z^2 = \mathbf{1} \qquad XZ = -ZX$$

- Mathematically: Projective $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

Projective symmetry is intrinsically quantum!

- Consequence: Every symmetric Hamiltonian has degeneracies

The Unexpected Power of Symmetry

This talk: Overview unexpected consequences of symmetry
intrinsic to quantum systems

Symmetry principles enforcing nontrivial quantum phenomena

The Unexpected Power of Symmetry

This talk: Overview unexpected consequences of symmetry
intrinsic to quantum systems

Symmetry principles enforcing nontrivial quantum phenomena

1) Correlated quantum matter/field theory

Symmetry can constraint low-energy dynamics

2) Quantum entanglement

Symmetry can enforce nontrivial entanglement

3) Generalized symmetry

New symmetry, same consequences

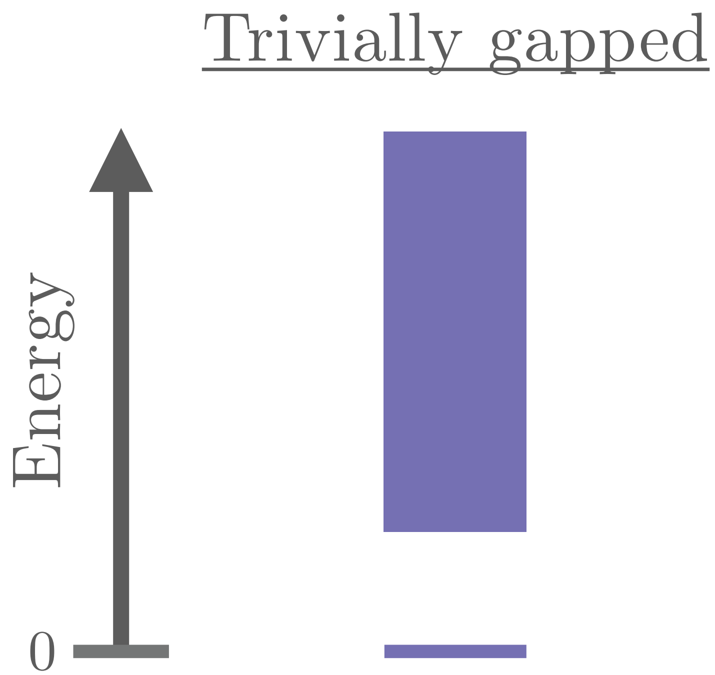
Correlated Quantum Matter/Field Theory

The Grand Challenge

Given a quantum system with many interacting degrees of freedom, what is its low-energy/long-distance behavior?

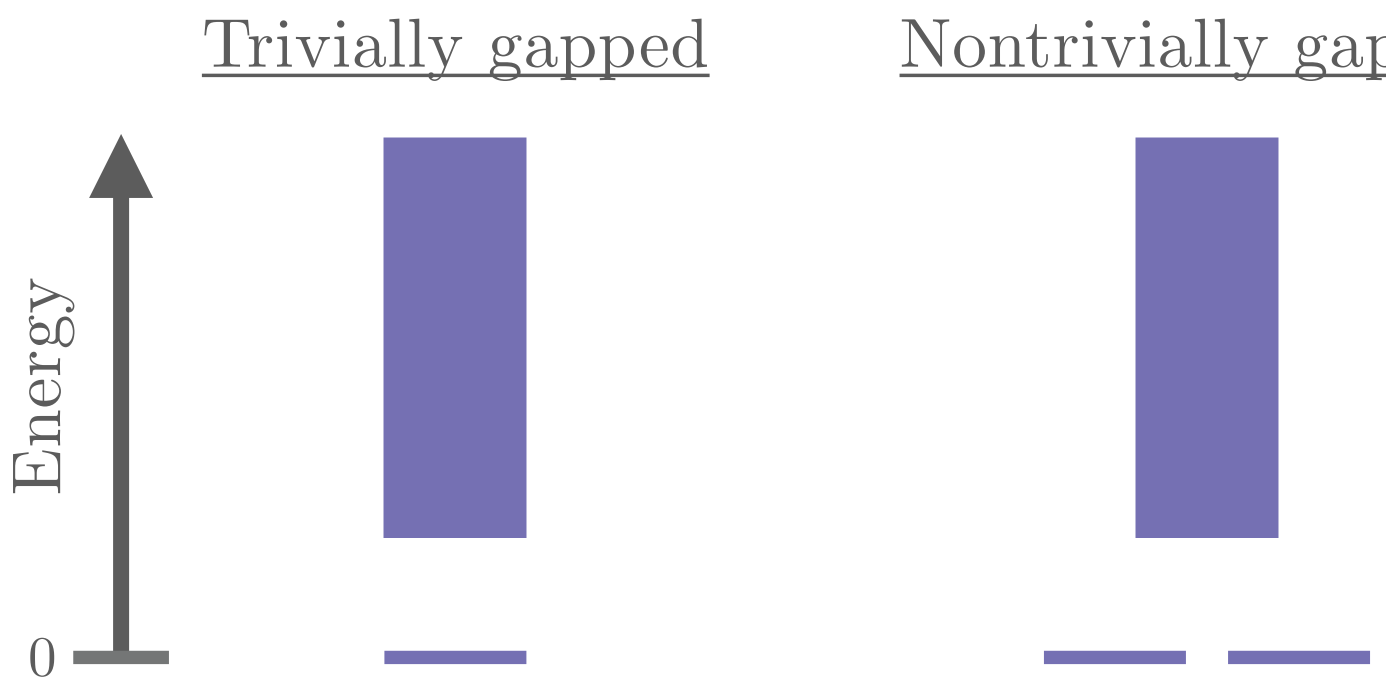
The Grand Challenge

Given a quantum system with many interacting degrees of freedom, what is its low-energy/long-distance behavior?



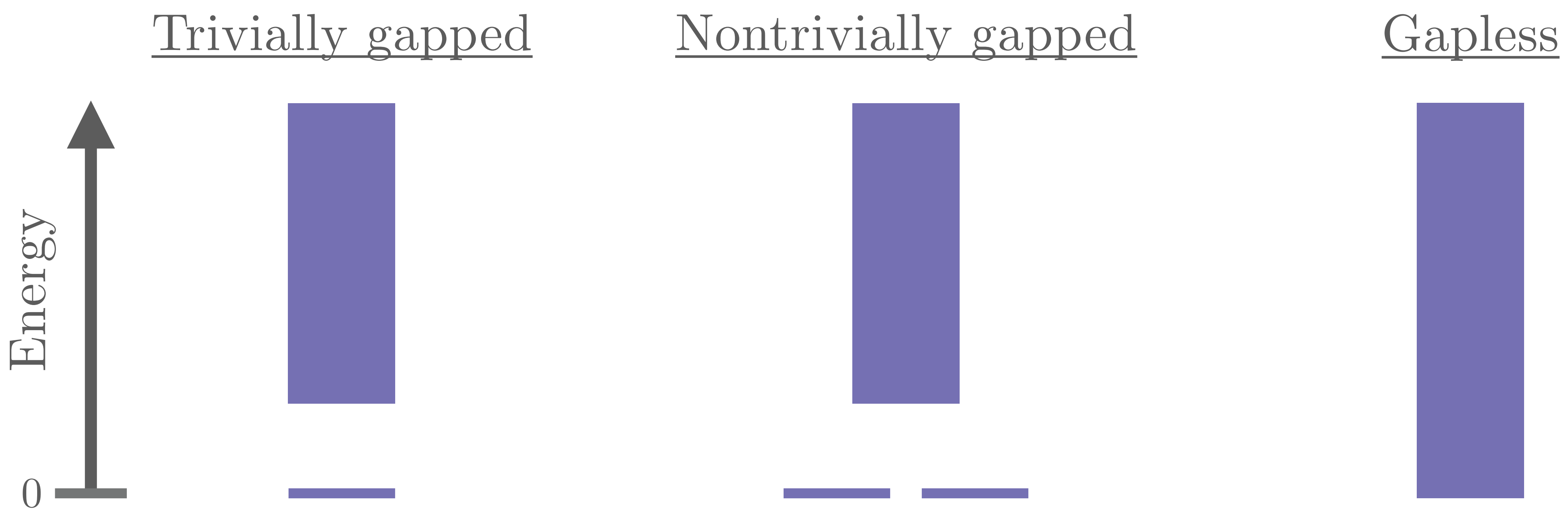
The Grand Challenge

Given a quantum system with many interacting degrees of freedom, what is its low-energy/long-distance behavior?



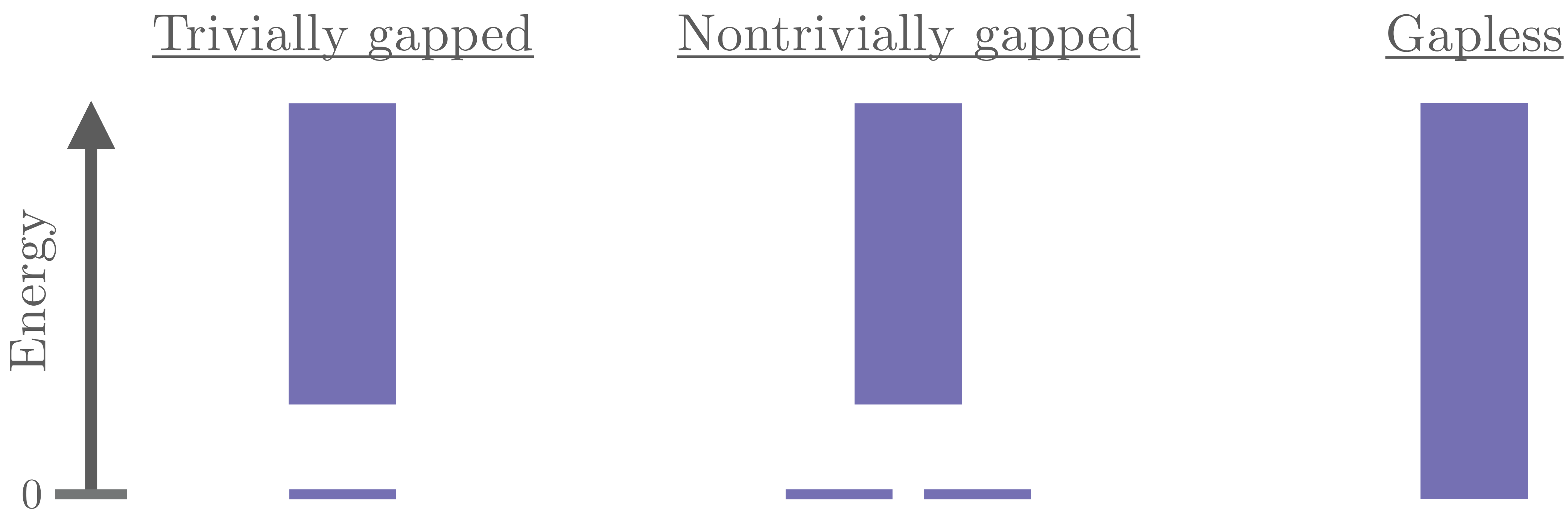
The Grand Challenge

Given a quantum system with many interacting degrees of freedom, what is its low-energy/long-distance behavior?



The Grand Challenge

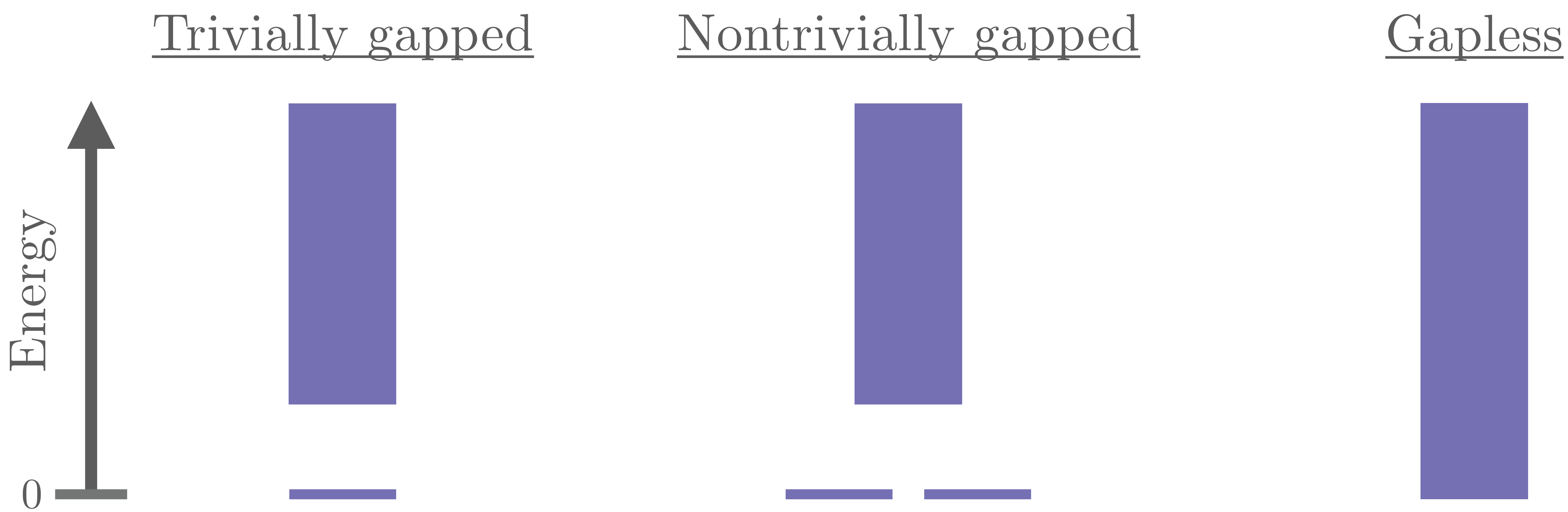
Given a quantum system with many interacting degrees of freedom, what is its low-energy/long-distance behavior?



- Notoriously difficult: strong interactions often defy conventional perturbative methods

The Grand Challenge

Given a quantum system with many interacting degrees of freedom, what is its low-energy/long-distance behavior?

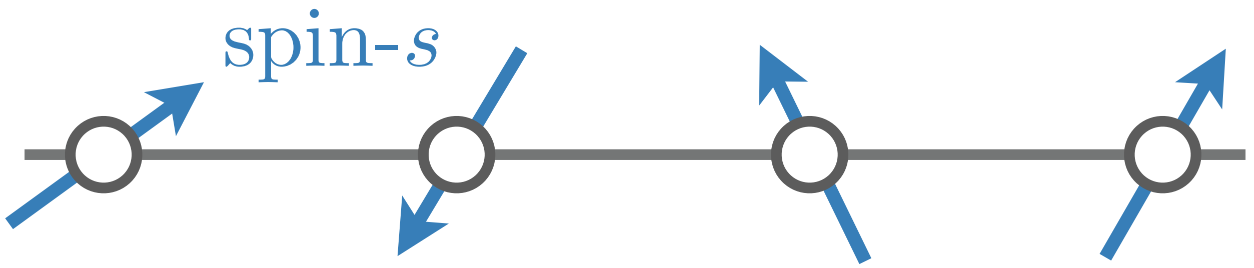


- Notoriously difficult: strong interactions often defy conventional perturbative methods

Symmetry can constrain the low-energy dynamics and organize distinct possibilities

The Lieb-Schultz-Mattis Theorem

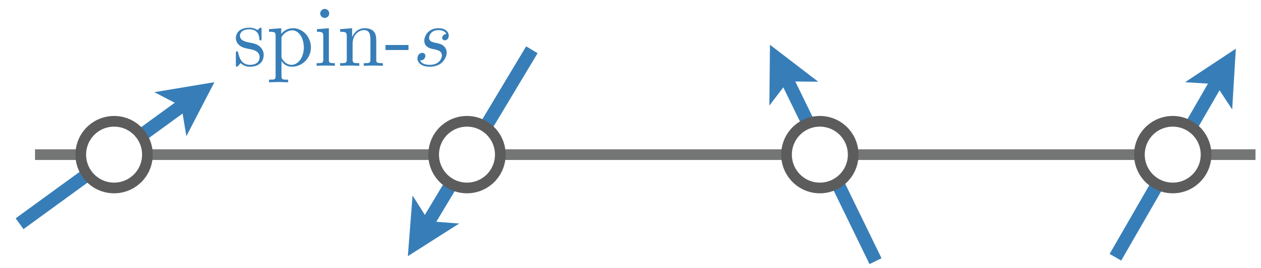
Consider Quantum spin chain



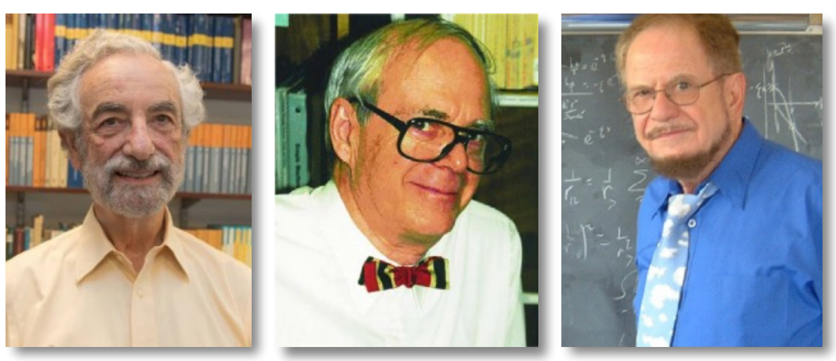
- Symmetry: $\vec{S}_j \mapsto \vec{S}_{j+1}$ and $\vec{S}_j \mapsto R\,\vec{S}_j$ with $R \in \mathrm{SO}(3)$

The Lieb-Schultz-Mattis Theorem

Consider Quantum spin chain



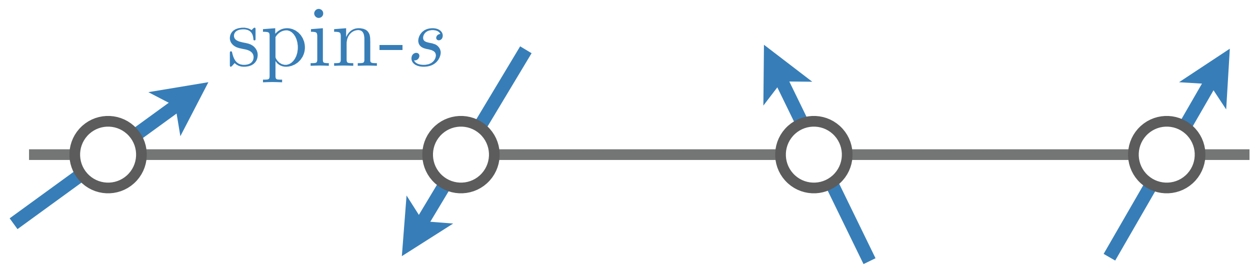
➤ Symmetry: $\vec{S}_j \mapsto \vec{S}_{j+1}$ and $\vec{S}_j \mapsto R\,\vec{S}_j$ with $R \in \mathrm{SO}(3)$



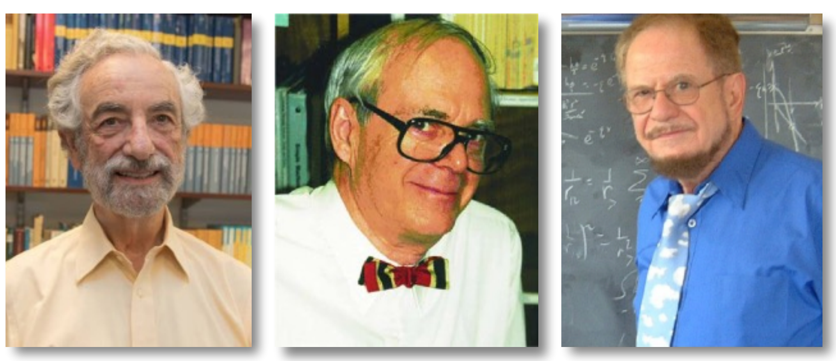
Lieb-Schultz-Mattis 1961: This symmetry forbids a trivial gapped phase when s is half-integer

The Lieb-Schultz-Mattis Theorem

Consider Quantum spin chain

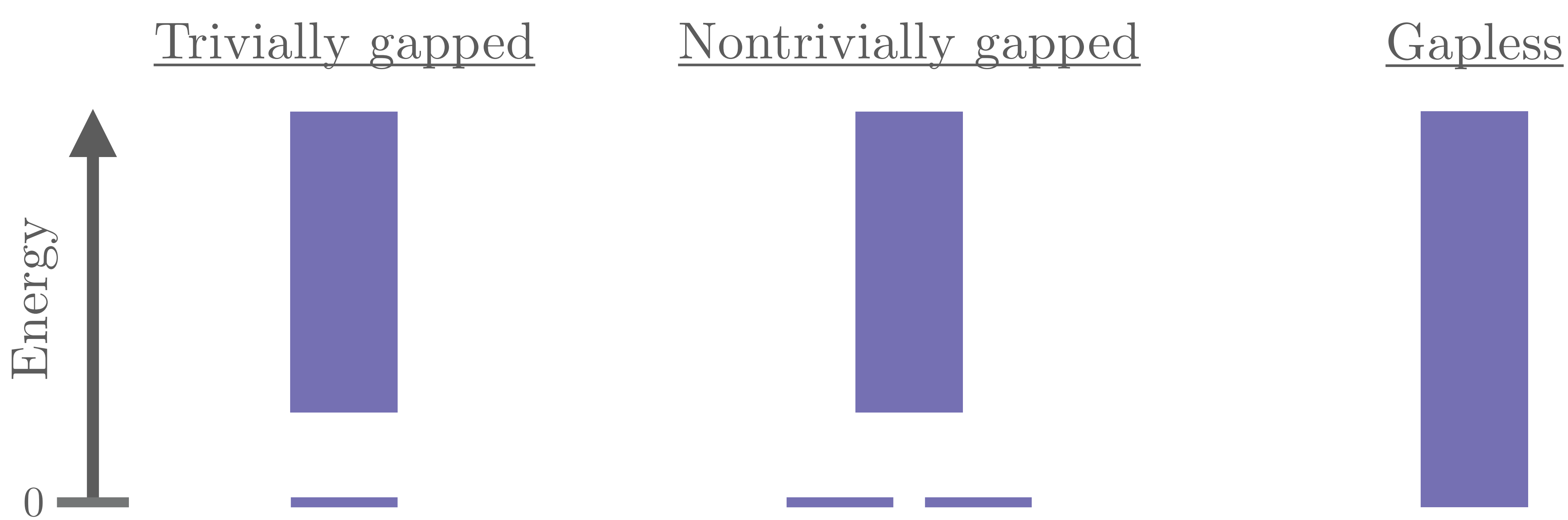


➤ Symmetry: $\vec{S}_j \mapsto \vec{S}_{j+1}$ and $\vec{S}_j \mapsto R\,\vec{S}_j$ with $R \in \mathrm{SO}(3)$



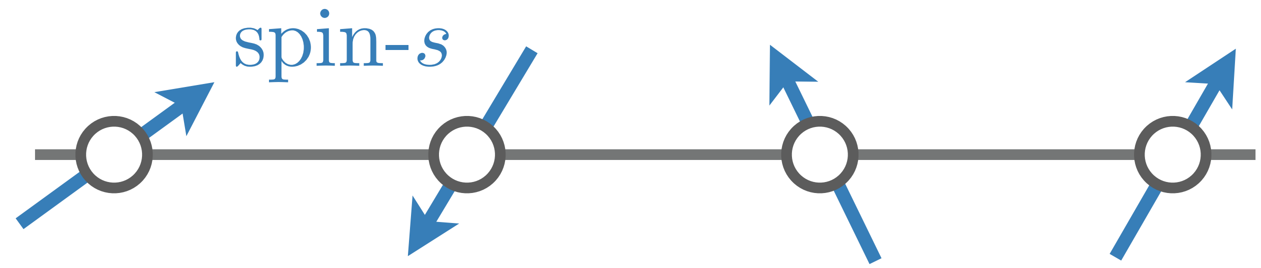
Lieb-Schultz-Mattis 1961: This symmetry forbids a trivial gapped phase when s is half-integer

if $s = 1, 2, 3, \cdots$

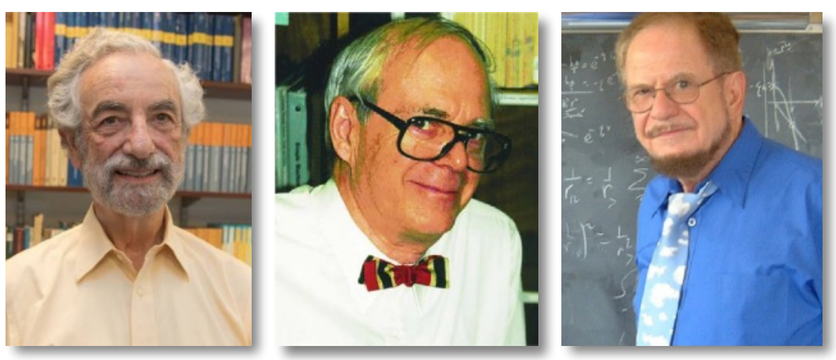


The Lieb-Schultz-Mattis Theorem

Consider Quantum spin chain

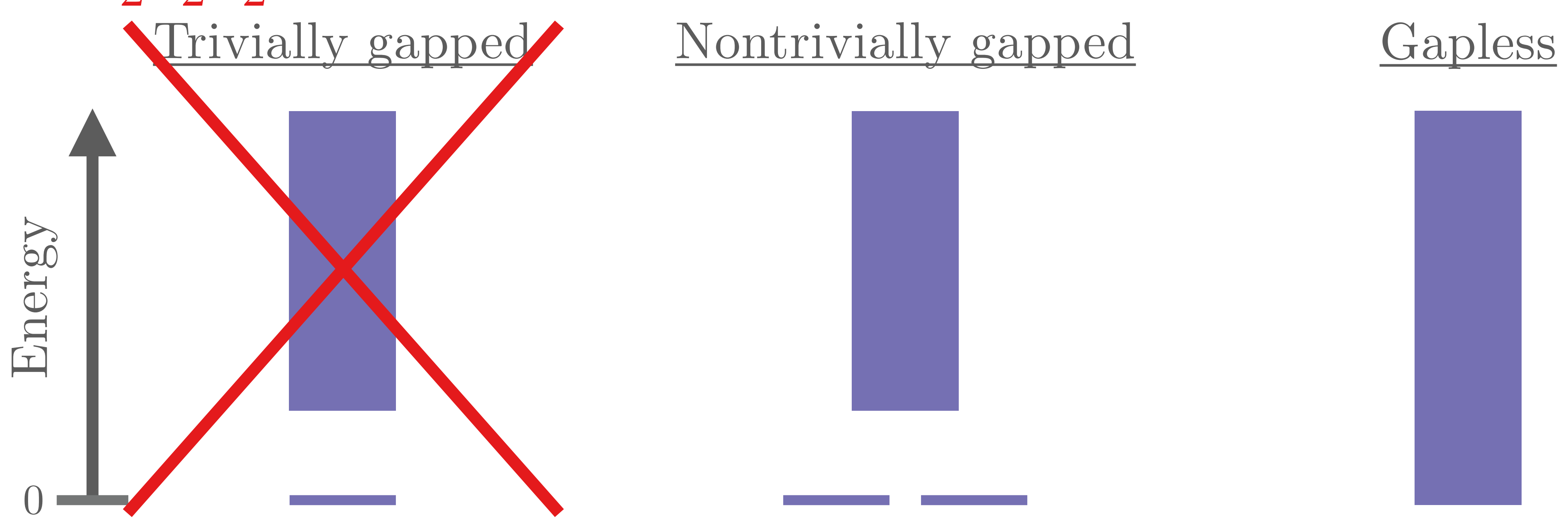


- Symmetry: $\vec{S}_j \mapsto \vec{S}_{j+1}$ and $\vec{S}_j \mapsto R\,\vec{S}_j$ with $R \in \mathrm{SO}(3)$



Lieb-Schultz-Mattis 1961: This symmetry forbids a trivial gapped phase when s is half-integer

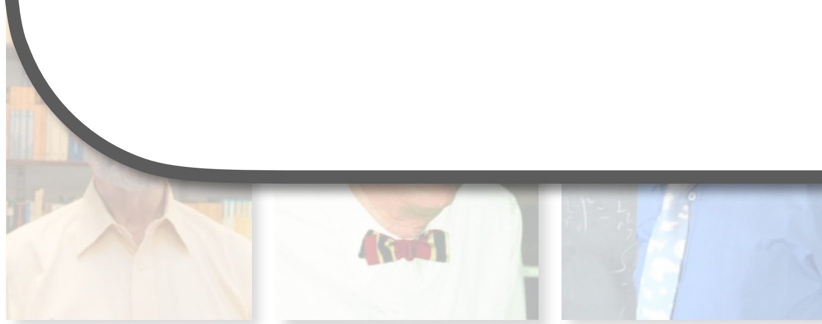
if $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$



The Lieb-Schultz-Mattis Theorem

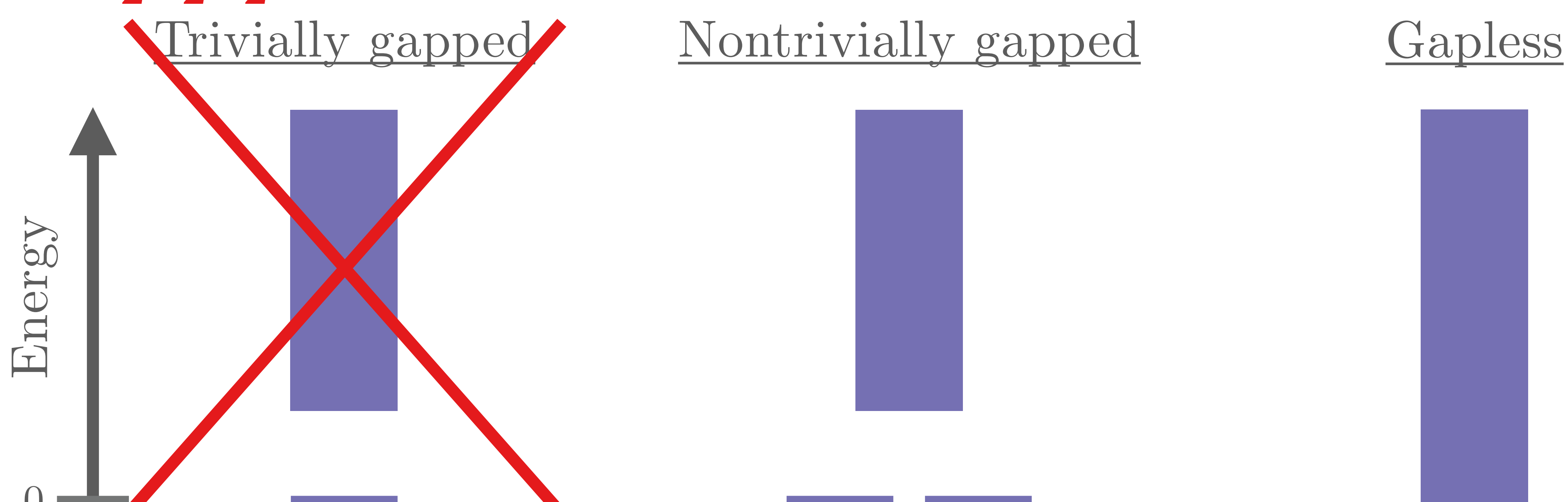
Consider Quantum spin chain

Holds for all symmetric Hamiltonians!



half-integer

if $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$

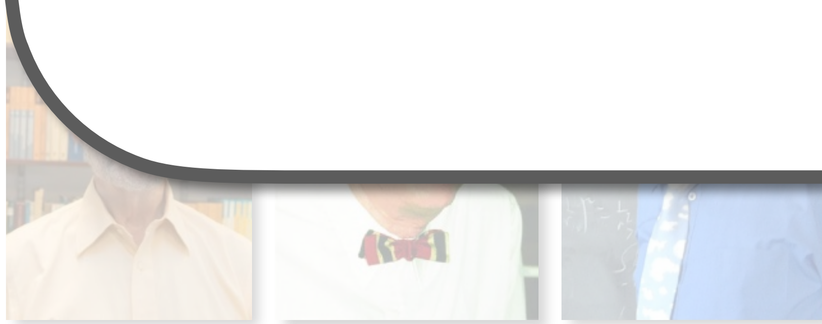


The Lieb-Schultz-Mattis Theorem

Consider Quantum spin chain

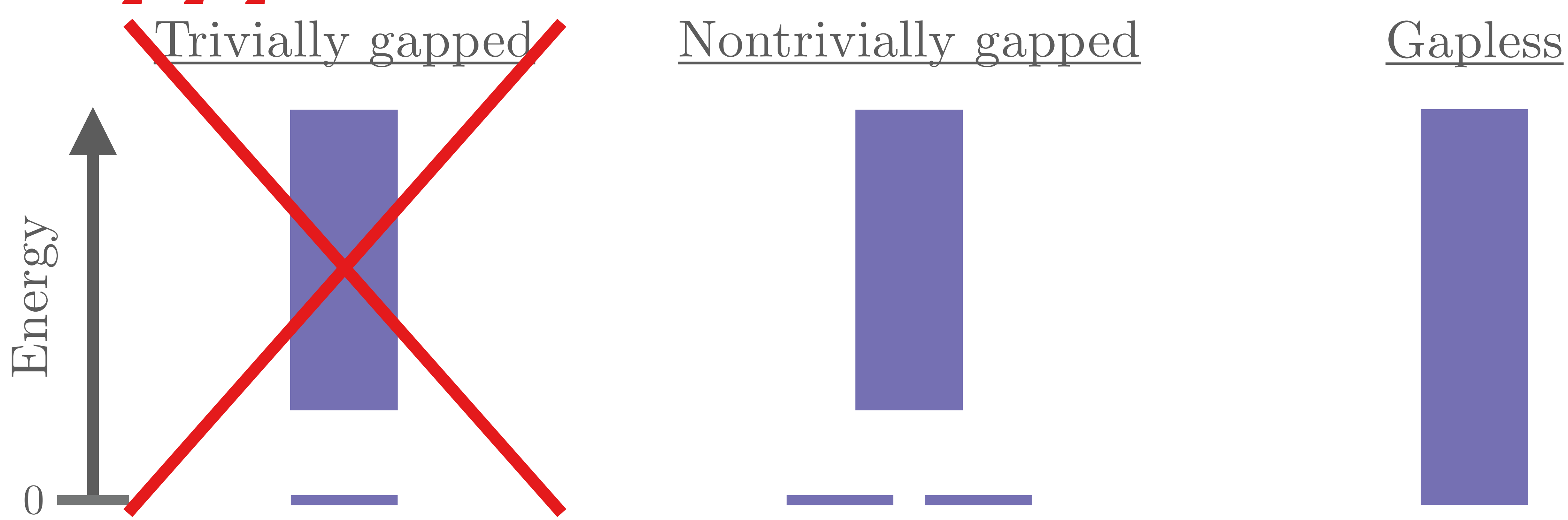
Holds for all symmetric Hamiltonians!

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$



half-integer

if $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$

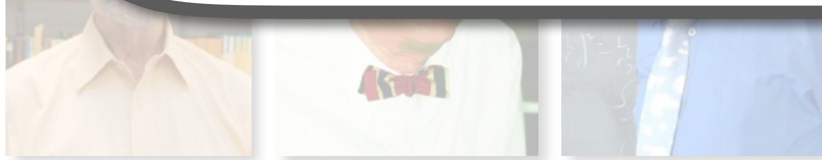


The Lieb-Schultz-Mattis Theorem

Consider Quantum spin chain

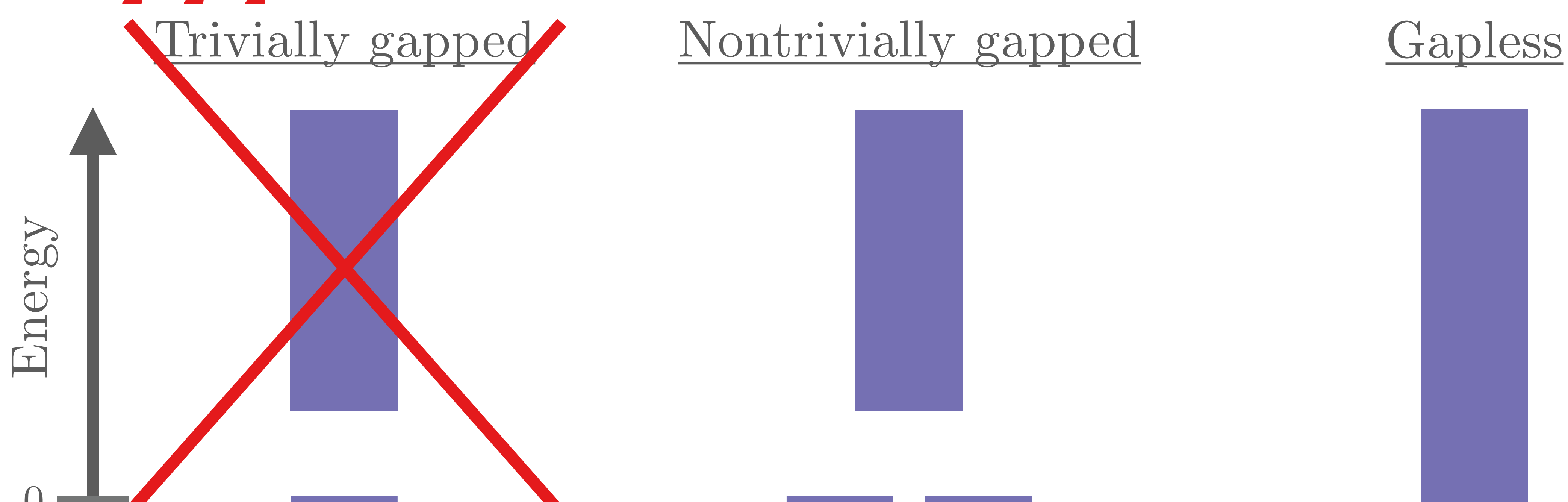
Holds for all symmetric Hamiltonians!

$$H = \sum_j \Big(\vec{S}_j \cdot \vec{S}_{j+1} + 67\, \vec{S}_j \cdot \vec{S}_{j+2} - 42\, (\vec{S}_j \cdot \vec{S}_{j+1})(\vec{S}_{j+1} \cdot \vec{S}_{j+2}) \Big)$$



half-integer

if $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$

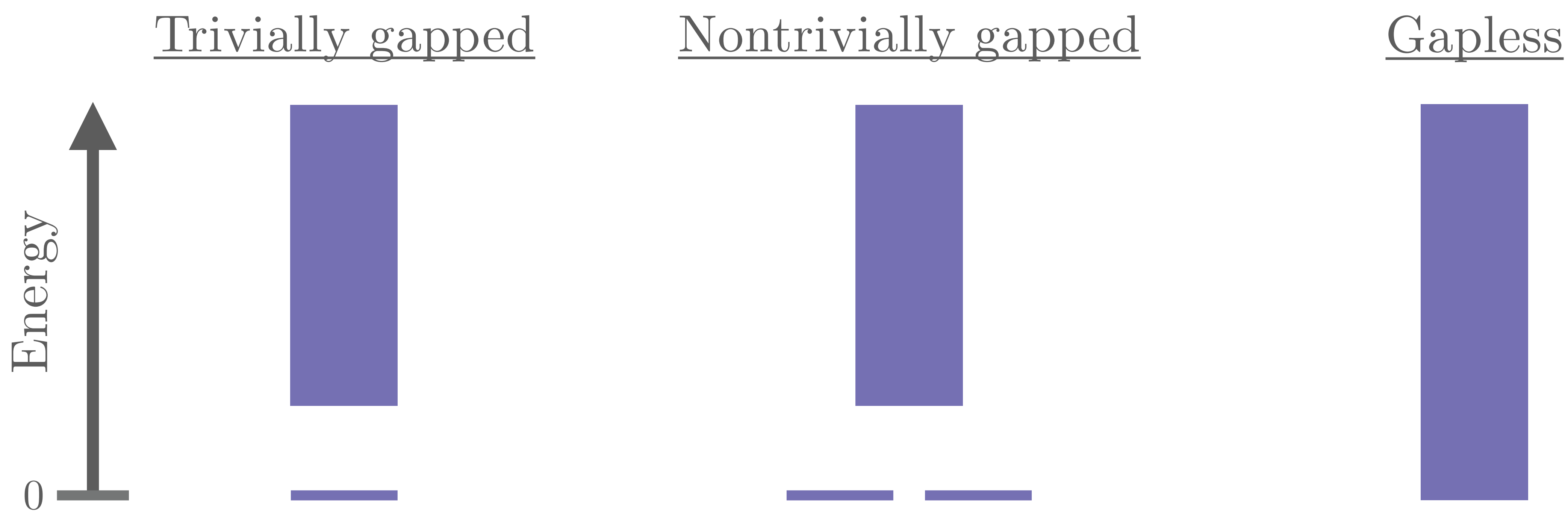


Symmetry-Enforced Gaplessness

Sometimes, symmetry is even more powerful

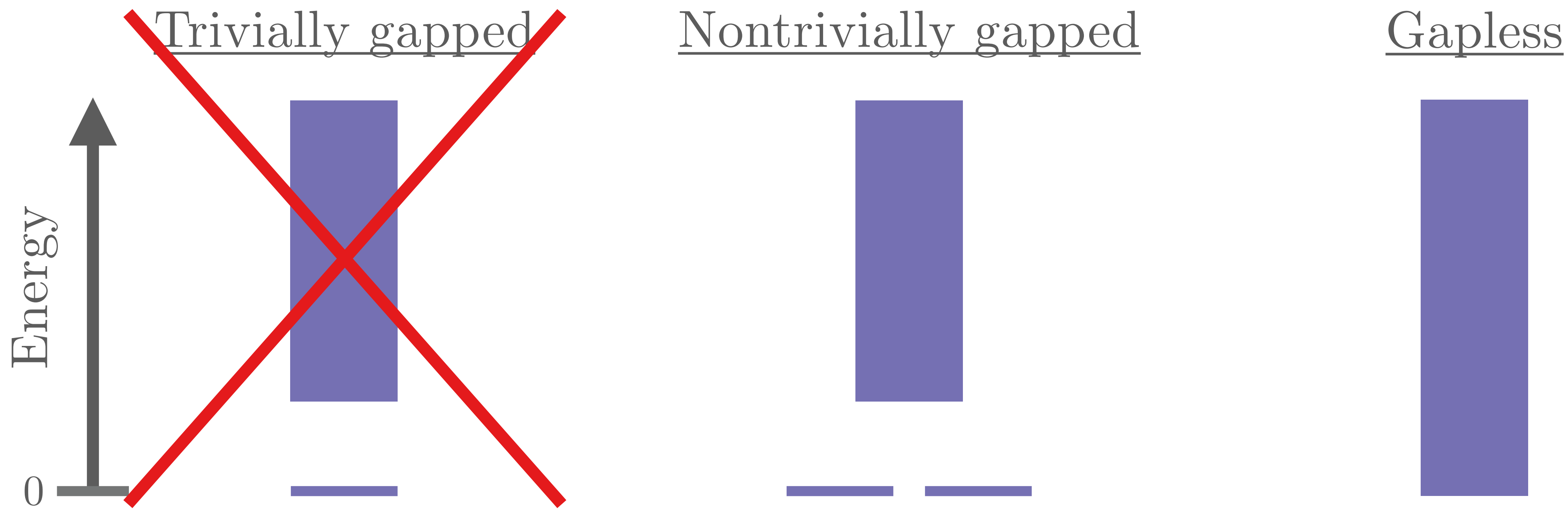
Symmetry-Enforced Gaplessness

Sometimes, symmetry is even more powerful



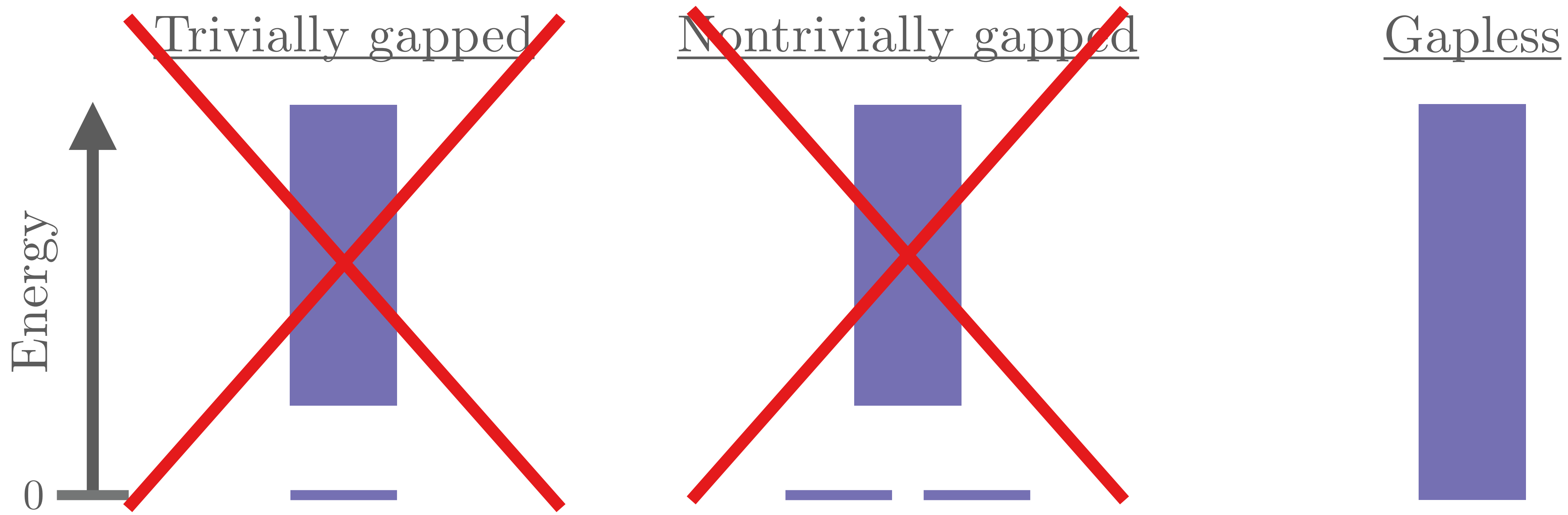
Symmetry-Enforced Gaplessness

Sometimes, symmetry is even more powerful



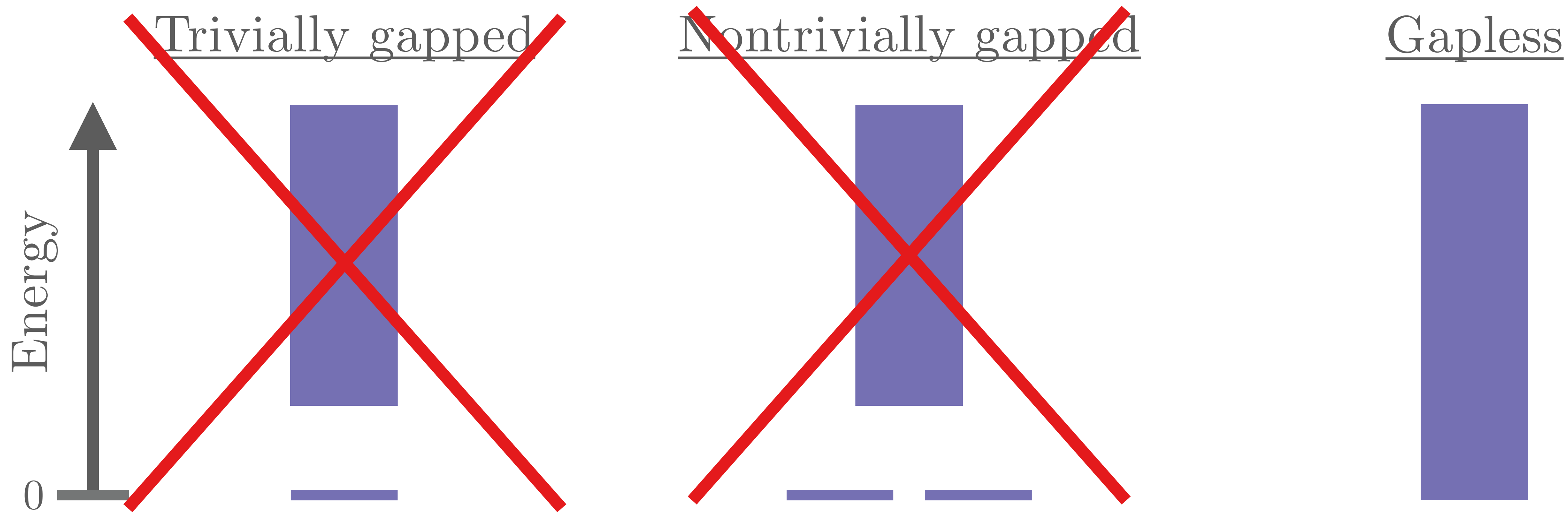
Symmetry-Enforced Gaplessness

Sometimes, symmetry is even more powerful



Symmetry-Enforced Gaplessness

Sometimes, symmetry is even more powerful



Symmetry-enforced gaplessness

Condensed Matter


Metlitski


Senthil


Wang


Xu

High Energy


Córdova


Lam


Ohmori


Tachikawa

Two Types of Gaplessness

Not all gaplessness is equal

Two Types of Gaplessness

Not all gaplessness is equal

Finite gaplessness

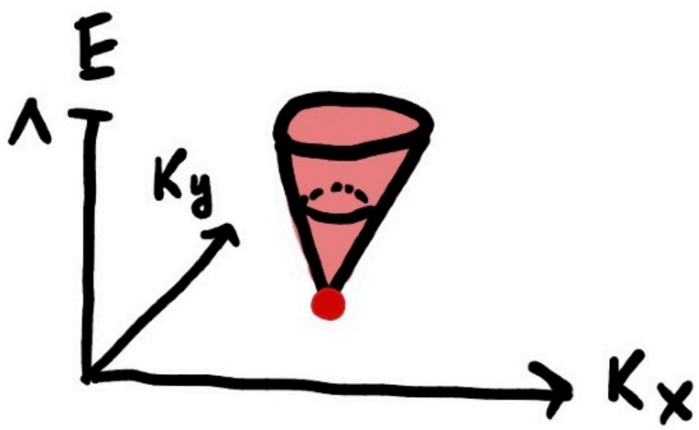
Infinite gaplessness

Two Types of Gaplessness

Not all gaplessness is equal

Finite gaplessness

Infinite gaplessness

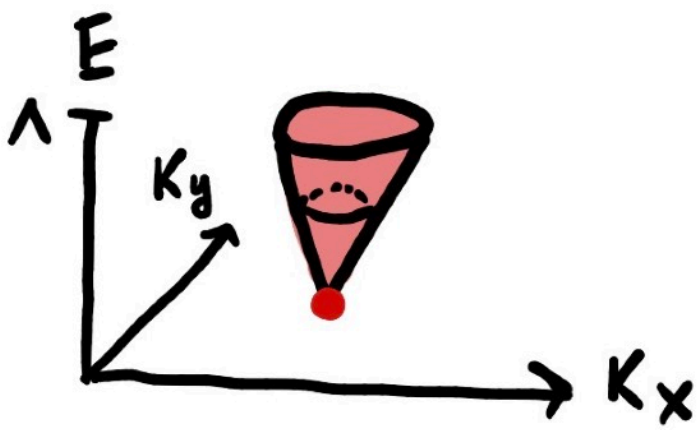


(within local QFT)

Two Types of Gaplessness

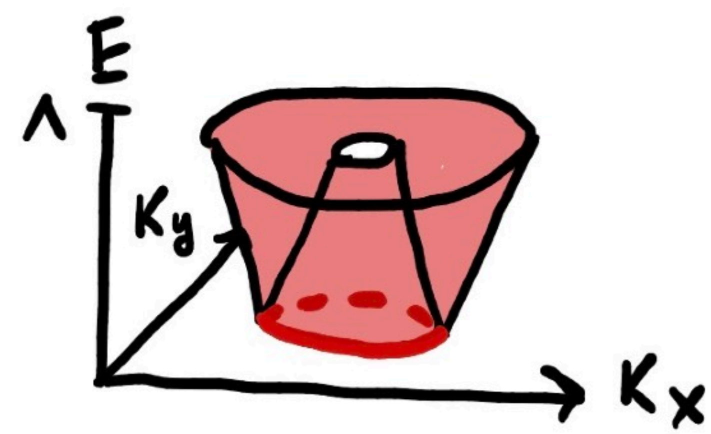
Not all gaplessness is equal

Finite gaplessness



(within local QFT)

Infinite gaplessness

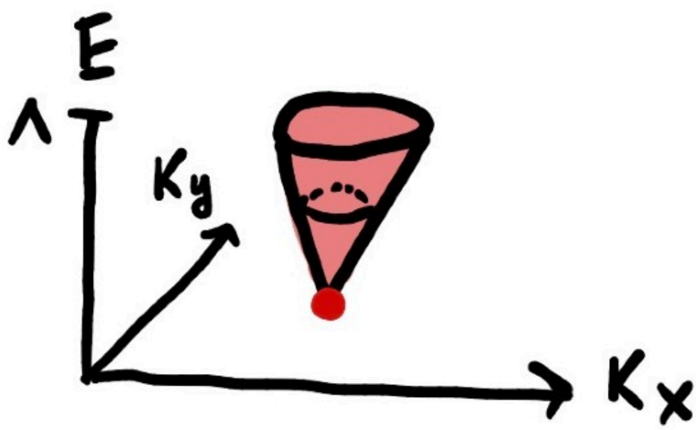


(beyond local QFT)

Two Types of Gaplessness

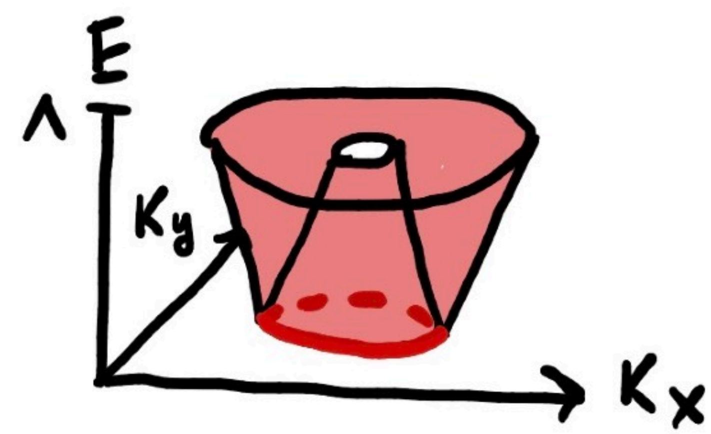
Not all gaplessness is equal

Finite gaplessness



(within local QFT)

Infinite gaplessness



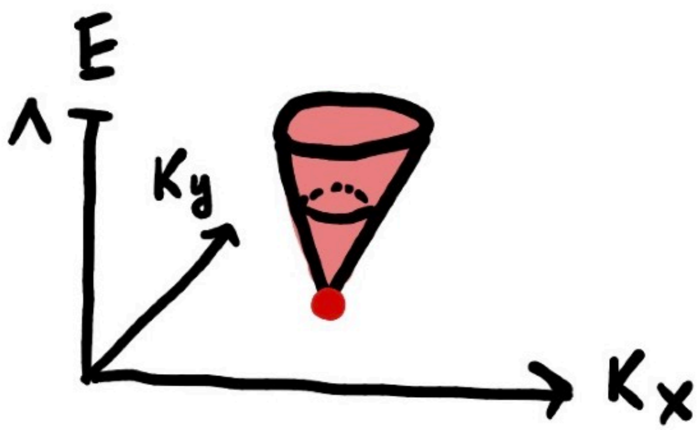
(beyond local QFT)

Question: Can a symmetry enforce infinite gaplessness?

Two Types of Gaplessness

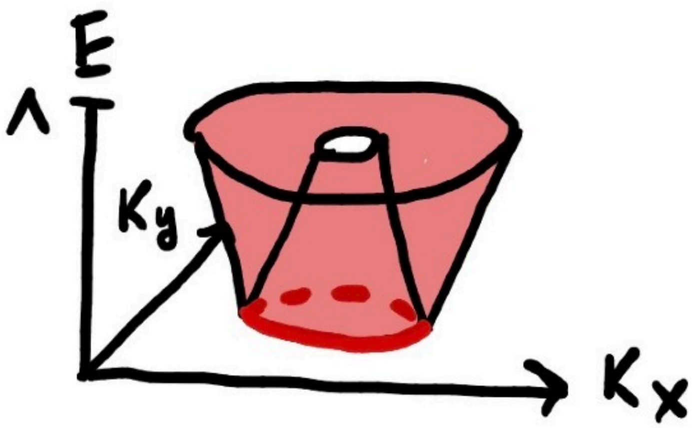
Not all gaplessness is equal

Finite gaplessness



(within local QFT)

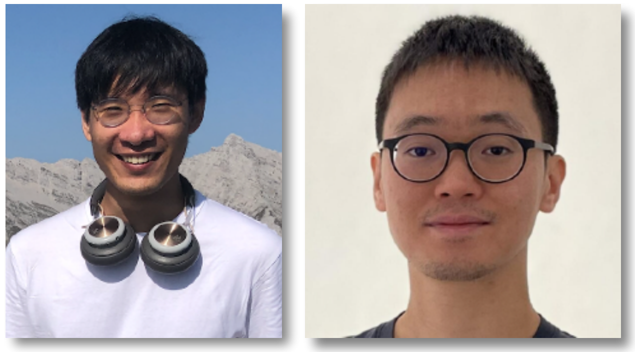
Infinite gaplessness



(beyond local QFT)

Question: Can a symmetry enforce infinite gaplessness?

Answer: Yes, symmetry-enforced Fermi surfaces!

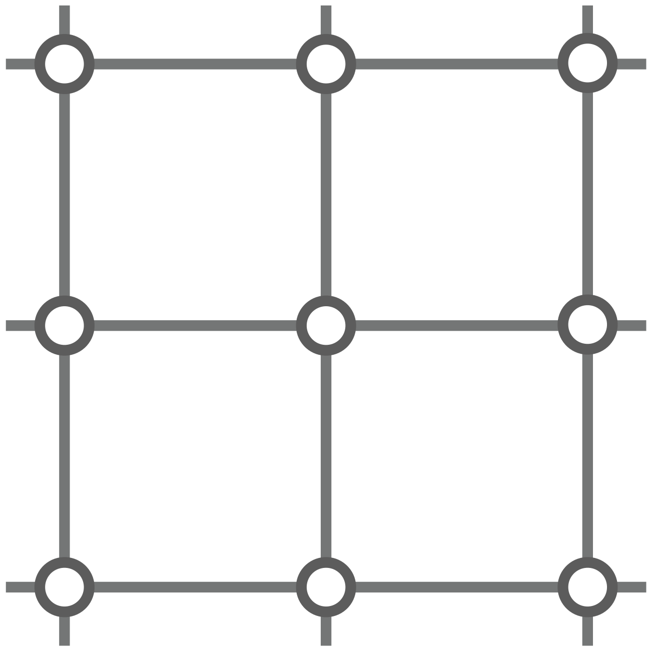


[Kim-**SP**-Shao, PRL '26]

Symmetry-Enforced Fermi surface

2D lattice with a fermion c_r on each site r

$$\{c_r, c^\dagger_{r'}\} = \delta_{r,r'} \qquad \{c_r, c_{r'}\} = 0$$

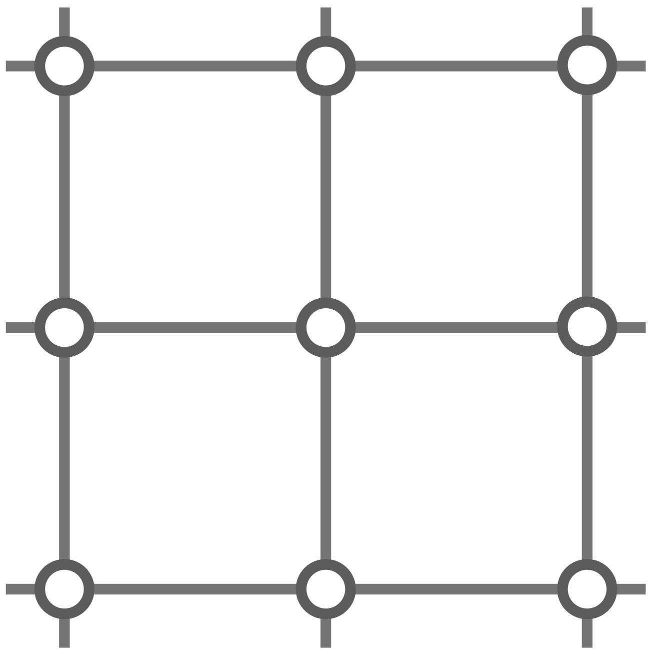


[Kim-**SP**-Shao, PRL '26]

Symmetry-Enforced Fermi surface

2D lattice with a fermion c_r on each site r

$$\{c_r, c_{r'}^\dagger\} = \delta_{r,r'} \qquad \{c_r, c_{r'}\} = 0$$



Be real 😎: $c_r = (a_r + \mathrm{i} b_r)/2$

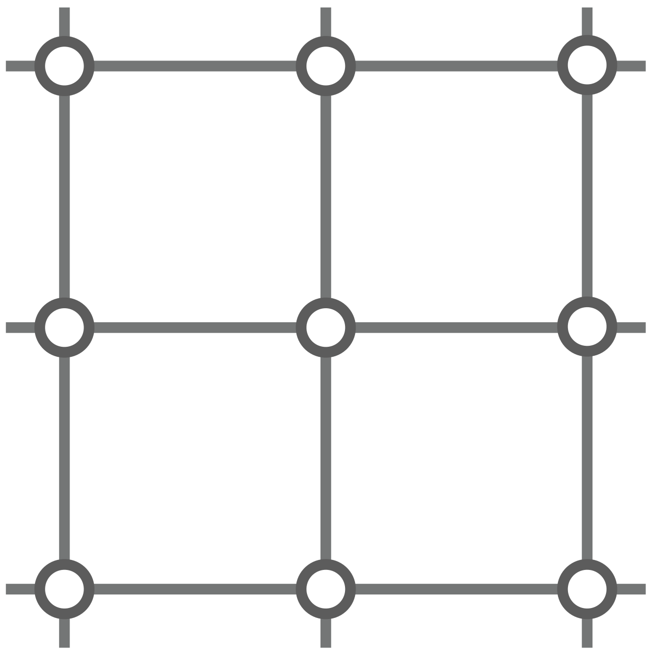
➤ Real (Majorana) fermions a_r and b_r

[Kim-**SP**-Shao, PRL '26]

Symmetry-Enforced Fermi surface

2D lattice with a fermion c_r on each site r

$$\{c_r, c^\dagger_{r'}\} = \delta_{r,r'} \qquad \{c_r, c_{r'}\} = 0$$



Be real 😎: $c_r = (a_r + \mathrm{i} b_r)/2$

- Real (Majorana) fermions a_r and b_r

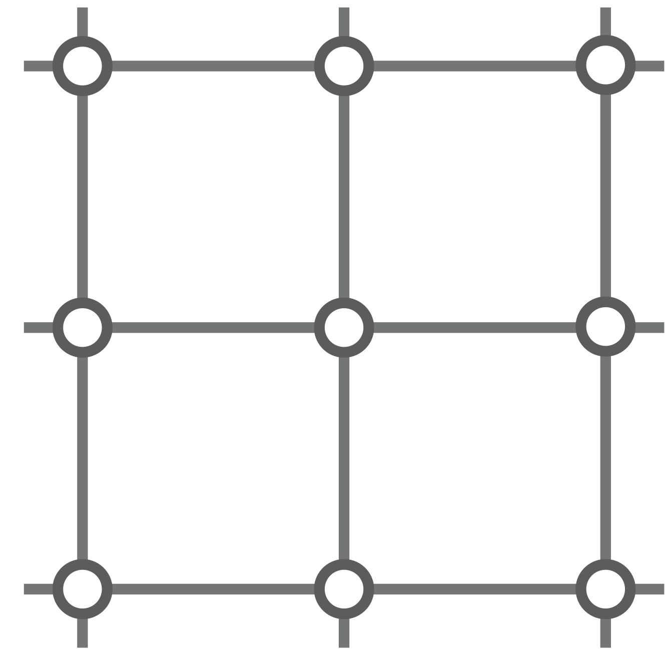
Symmetry:

1. Fermion number symmetry: $c_r \mapsto \mathrm{e}^{-\mathrm{i}\theta} c_r$
2. Majorana translations: $a_r, b_r \mapsto a_r, b_{r+v}$

[Kim-**SP**-Shao, PRL '26]

Symmetry-Enforced Fermi surface

2D lattice with a fermion c_r on each site r



$$\{c_r, c^\dagger_{r'}\} = \delta_{r,r'} \qquad \{c_r, c_{r'}\} = 0$$

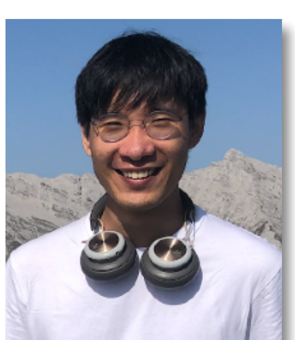
Be real 😎: $c_r = (a_r + \mathrm{i} b_r)/2$

➤ Real (Majorana) fermions a_r and b_r

Symmetry:

1. Fermion number symmetry: $c_r \mapsto \mathrm{e}^{-\mathrm{i}\theta} c_r$
2. Majorana translations: $a_r, b_r \mapsto a_r, b_{r+v}$

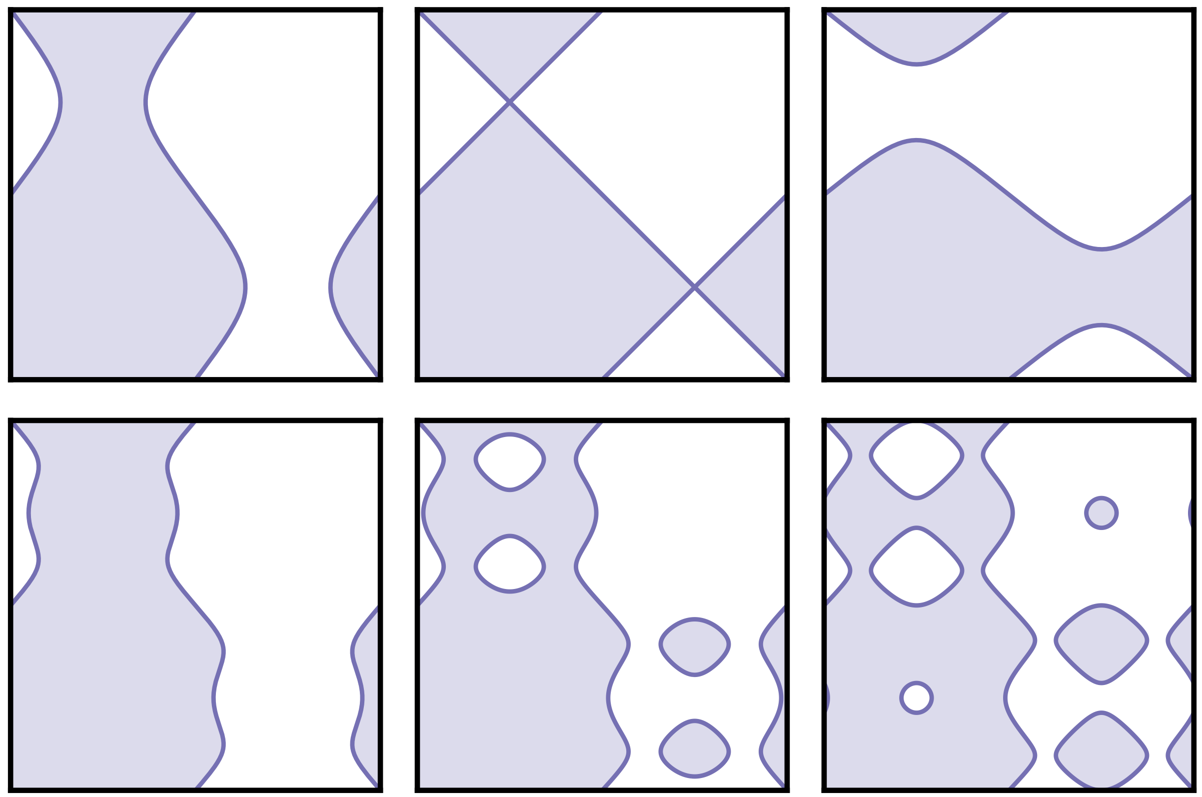
➤ Every symmetric Hamiltonian has a Fermi surface



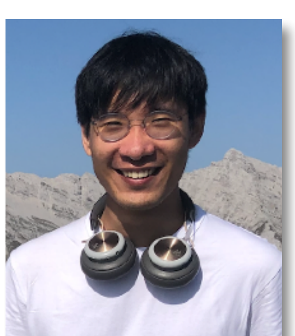
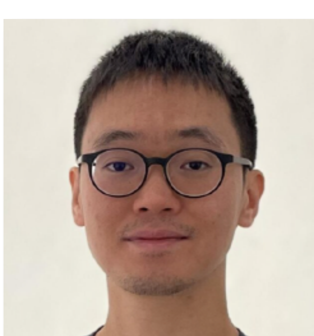
[Kim-**SP**-Shao, PRL '26]

Symmetry-Enforced Fermi surface

These symmetry-enforced Fermi surfaces are topologically nontrivial: exhibit non-contractible components



➤ Every symmetric Hamiltonian has a Fermi surface

[Kim-**SP**-Shao, PRL ’26]

Onsager Algebra

Fermion number and Majorana translation symmetry give rise to conserved operators

$$Q_v = \frac{\mathrm{i}}{2} \sum_r a_r b_{r+v} \qquad G_v = \frac{\mathrm{i}}{2} \sum_r (a_r a_{r+v} - b_r b_{r+v})$$

Onsager Algebra

Fermion number and Majorana translation symmetry give rise to conserved operators

$$Q_v = \frac{\mathrm{i}}{2}\sum_r a_r b_{r+v} \qquad G_v = \frac{\mathrm{i}}{2}\sum_r (a_r a_{r+v} - b_r b_{r+v})$$

- Form the Onsager algebra

1944

Onsager Algebra

Fermion number and Majorana translation symmetry give rise to conserved operators

$$Q_v = \frac{\mathrm{i}}{2}\sum_r a_r b_{r+v} \qquad G_v = \frac{\mathrm{i}}{2}\sum_r (a_r a_{r+v} - b_r b_{r+v})$$

- Form the Onsager algebra

1944

$$[Q_v, Q_{v'}] = \mathrm{i}G_{v'-v}$$
$$[G_v, G_{v'}] = 0$$
$$[Q_v, G_{v'}] = 2\mathrm{i}(Q_{v-v'} - Q_{v+v'})$$

Onsager Algebra

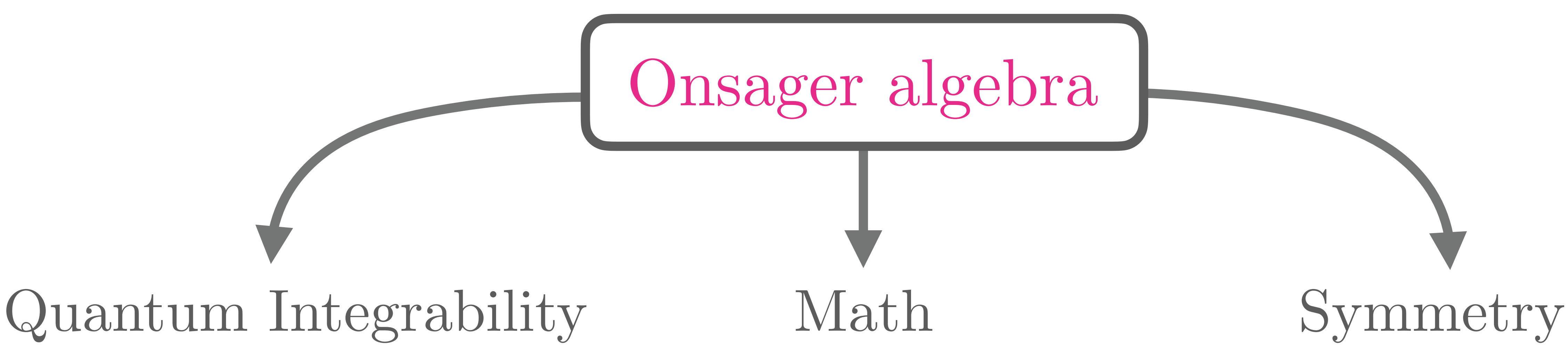
Fermion number and Majorana translation symmetry give rise to conserved operators

$$Q_v = \frac{\mathrm{i}}{2}\sum_r a_r b_{r+v} \qquad G_v = \frac{\mathrm{i}}{2}\sum_r (a_r a_{r+v} - b_r b_{r+v})$$

- Form the Onsager algebra

1944

$$[Q_v, Q_{v'}] = \mathrm{i}G_{v'-v}$$
$$[G_v, G_{v'}] = 0$$
$$[Q_v, G_{v'}] = 2\mathrm{i}(Q_{v-v'} - Q_{v+v'})$$



Gifts from Onsager

Constraint

Symmetry-enforced Fermi surfaces
[Kim-**SP**-Shao, PRL '26]

Gifts from Onsager

Constraint

Symmetry-enforced Fermi surfaces
[Kim-**SP**-Shao, PRL '26]

1+1d Gaplessness
[Chatterjee-**SP**-Shao, PRL '25
SP-Chatterjee-Shao, SciPost '25]

Symmetry-protected Weyl fermions
[Gioia-Thorngren, PRL '26]

Symmetry-enforced Dirac cones
[**SP**-Kim-Chatterjee-Shao, PRL '25]

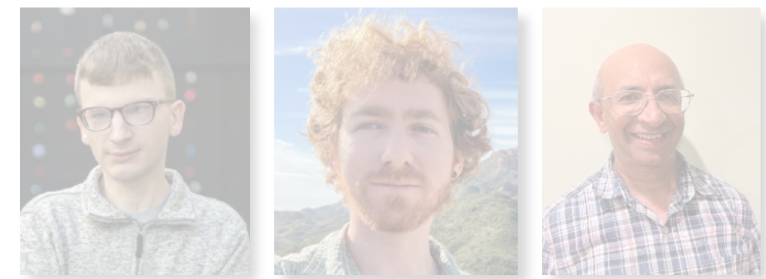
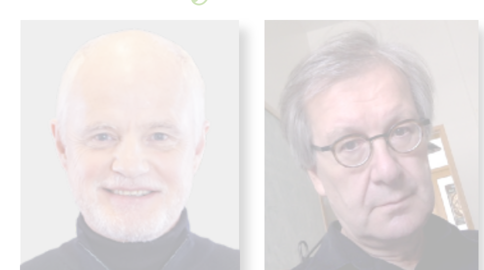
Gifts from Onsager

Constraint	't Hooft Anomaly
Symmetry-enforced Fermi surfaces [Kim-**SP**-Shao, PRL '26]	
1+1d Gaplessness [Chatterjee-**SP**-Shao, PRL '25 **SP**-Chatterjee-Shao, SciPost '25]	
Symmetry-protected Weyl fermions [Gioia-Thorngren, PRL '26]	
Symmetry-enforced Dirac cones [**SP**-Kim-Chatterjee-Shao, PRL '25]	

Gifts from Onsager

Constraint	't Hooft Anomaly
Symmetry-enforced Fermi surfaces [Kim-**SP**-Shao, PRL '26]	LU(1) anomaly   
1+1d Gaplessness [Chatterjee-**SP**-Shao, PRL '25 **SP**-Chatterjee-Shao, SciPost '25]	Schwinger anomaly 
Symmetry-protected Weyl fermions [Gioia-Thorngren, PRL '26]	Witten anomaly 
Symmetry-enforced Dirac cones [**SP**-Kim-Chatterjee-Shao, PRL '25]	Parity anomaly  

Gifts from Onsager

Constraint	't Hooft Anomaly
Symmetry-enforced Fermi surfaces [Kim-**SP**-Shao, PRL '26]	LU(1) anomaly 
1+1d Gaplessness [Chatterjee-**SP**-Shao, PRL '25 **SP**-Chatterjee-Shao, SciPost '25]	Schwinger anomaly 
Symmetry-protected Weyl fermions [Gioia-Thorngren, PRL '26]	Witten anomaly 
Symmetry-enforced Dirac cones [**SP**-Kim-Chatterjee-Shao, PRL '25]	Parity anomaly 

First Lesson

Symmetry in quantum systems can constrain the low-energy/long-distance behavior

Quantum Entanglement

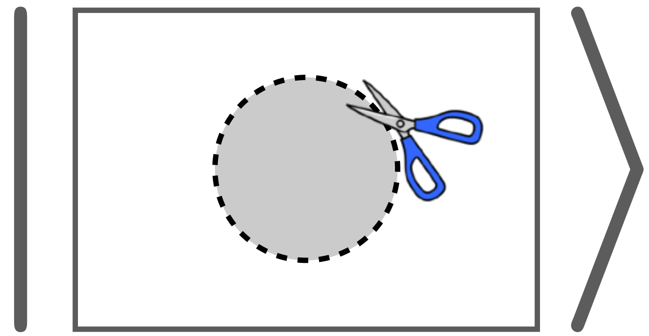
Entanglement Patterns

Given a quantum system with many interacting degrees of freedom, what is its ground state's spatial entanglement pattern?

Entanglement Patterns

Given a quantum system with many interacting degrees of freedom, what is its ground state's spatial entanglement pattern?

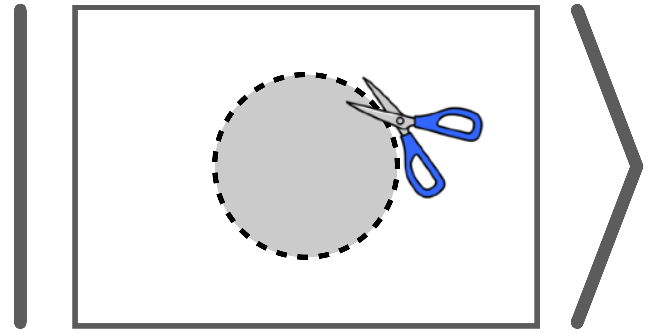
No entanglement



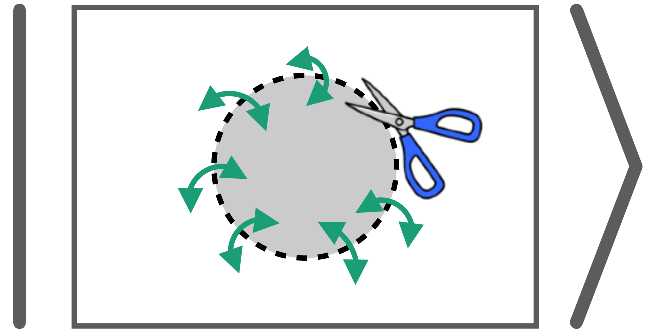
Entanglement Patterns

Given a quantum system with many interacting degrees of freedom, what is its ground state's spatial entanglement pattern?

No entanglement



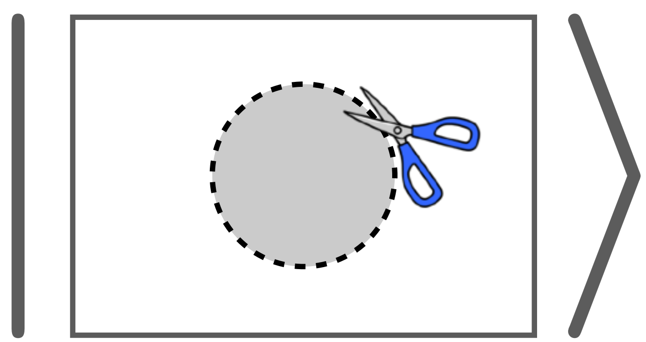
Short-range entanglement



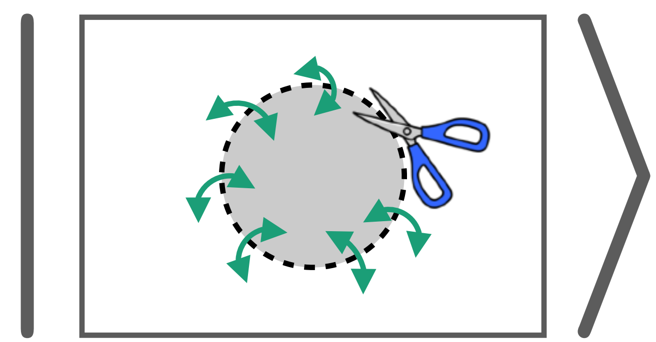
Entanglement Patterns

Given a quantum system with many interacting degrees of freedom, what is its ground state's spatial entanglement pattern?

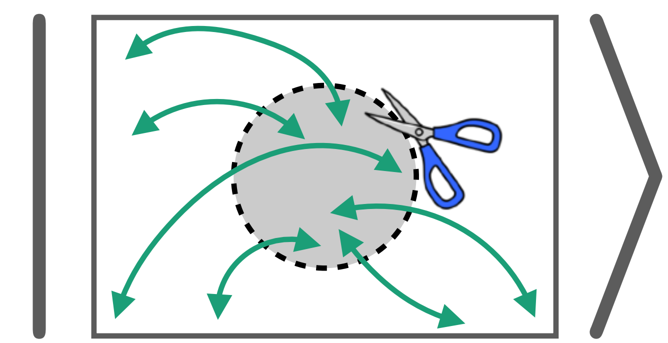
No entanglement



Short-range entanglement

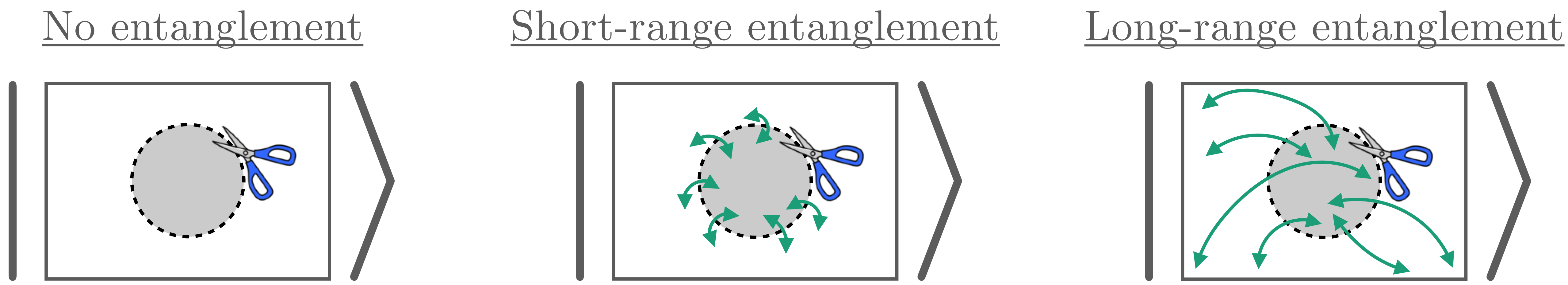


Long-range entanglement



Entanglement Patterns

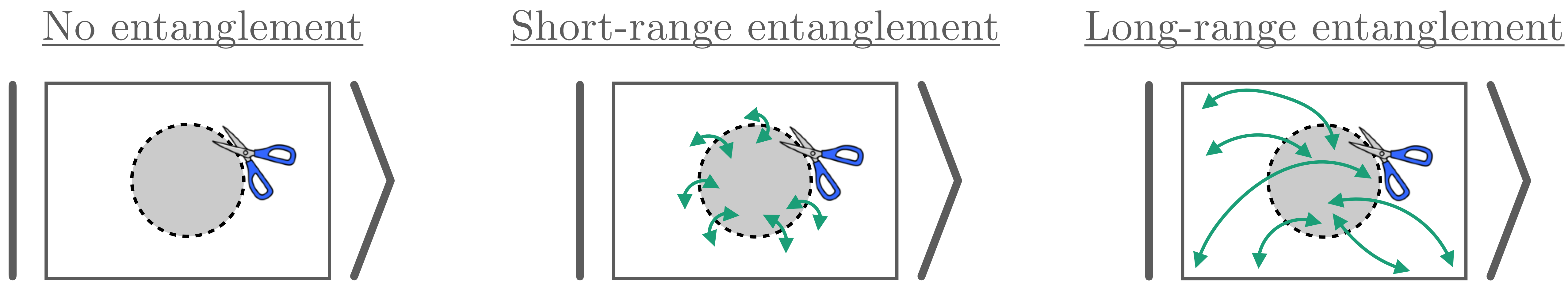
Given a quantum system with many interacting degrees of freedom, what is its ground state's spatial entanglement pattern?



➤ Notoriously difficult: Exponential complexity of the many body Hilbert space

Entanglement Patterns

Given a quantum system with many interacting degrees of freedom, what is its ground state's spatial entanglement pattern?



- ➤ Notoriously difficult: Exponential complexity of the many body Hilbert space

Symmetry can constrain the ground state entanglement and organize distinct possibilities

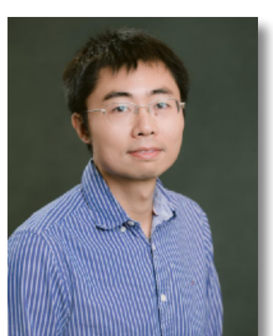
Symmetry-Enforced Entanglement

Symmetry can enforce states to have nontrivial entanglement

Interface of Condensed Matter and Quantum Information


Cheng


Else


Lu


Oshikawa


Ran

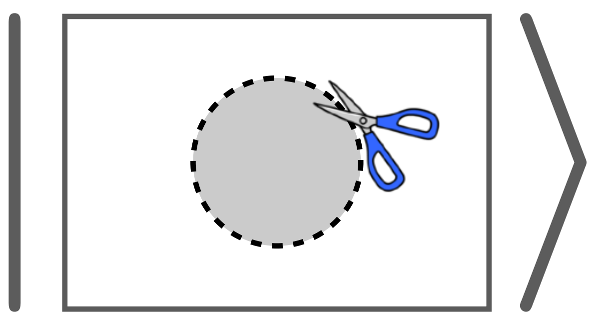

Thorngren


Vishwanath

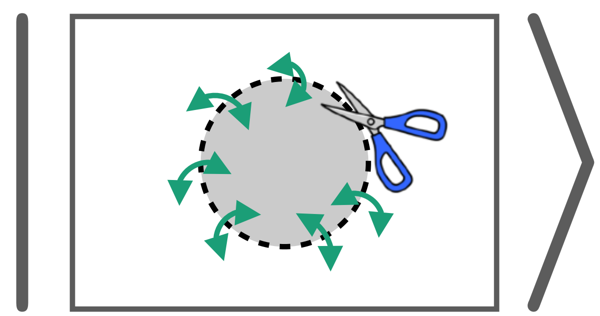
Symmetry-Enforced Entanglement

Symmetry can enforce states to have nontrivial entanglement

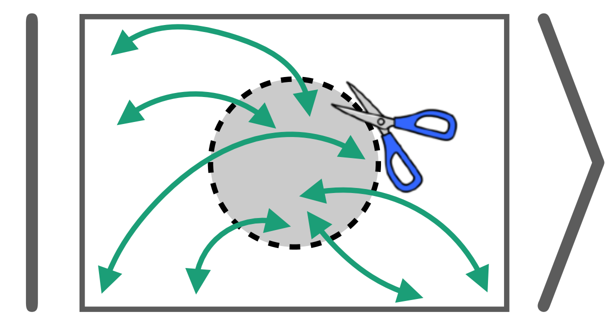
No entanglement



Short-range entanglement



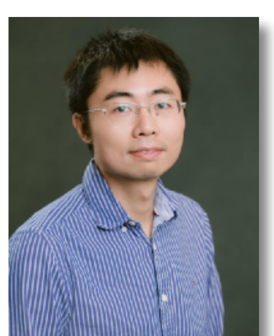
Long-range entanglement



Interface of Condensed Matter and Quantum Information

Cheng

Else



Lu

Oshikawa

Ran

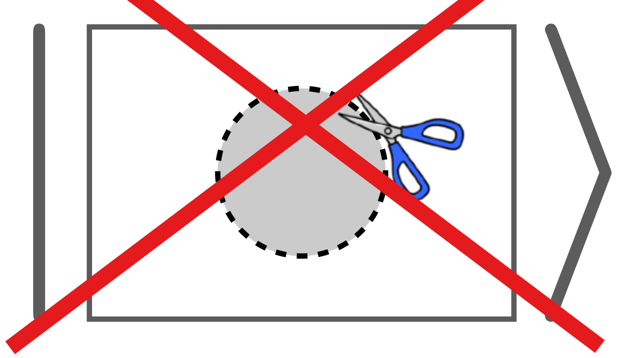
Thorngren

Vishwanath

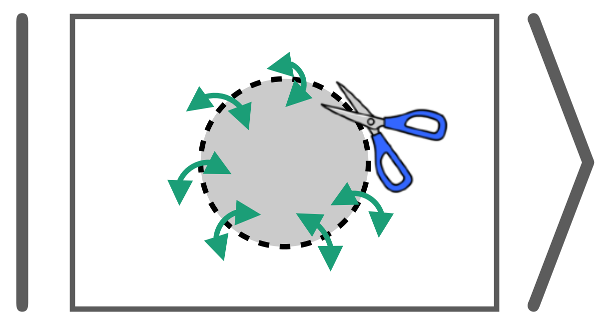
Symmetry-Enforced Entanglement

Symmetry can enforce states to have nontrivial entanglement

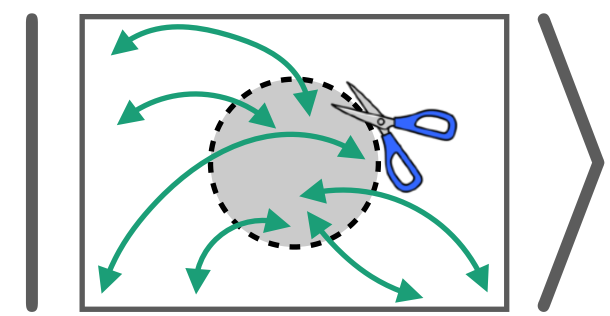
No entanglement



Short-range entanglement



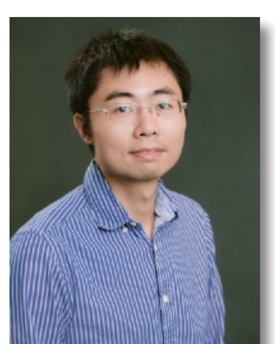
Long-range entanglement



Interface of Condensed Matter and Quantum Information


Cheng


Else


Lu


Oshikawa

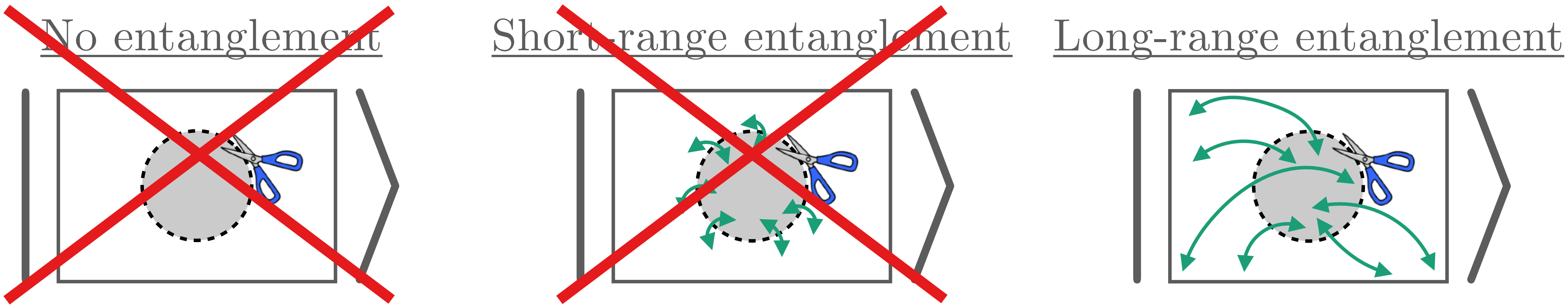

Ran


Thorngren

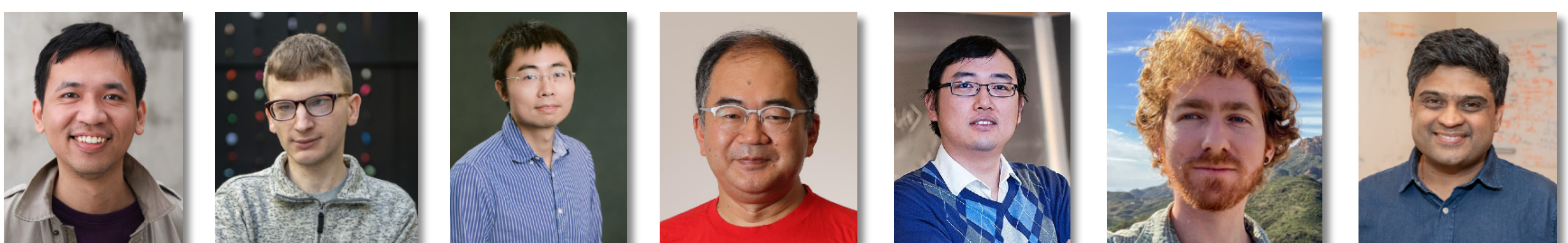

Vishwanath

Symmetry-Enforced Entanglement

Symmetry can enforce states to have nontrivial entanglement



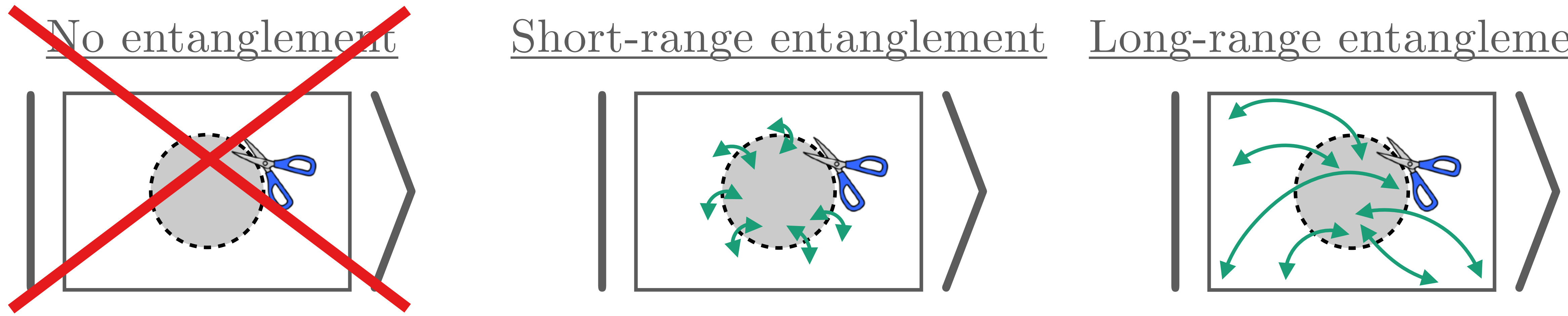
Interface of Condensed Matter and Quantum Information



Cheng Else Lu Oshikawa Ran Thorngren Vishwanath

Symmetry-Enforced Entanglement

Symmetry can enforce states to have nontrivial entanglement



Example: Locally projective symmetry

$$U = \prod_j U_j$$

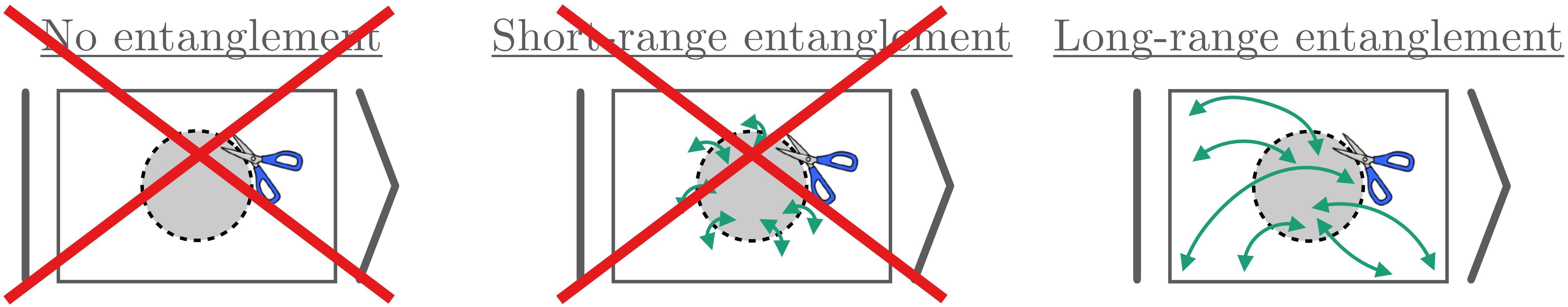
- Trivially entangled state: $|\psi\rangle = \bigotimes_j |\psi_j\rangle$

$$U|\psi\rangle = |\psi\rangle \implies U_j \text{ acts on } |\psi_j\rangle \text{ as a c-number:}$$

Impossible for projective U_j

Symmetry-Enforced Entanglement

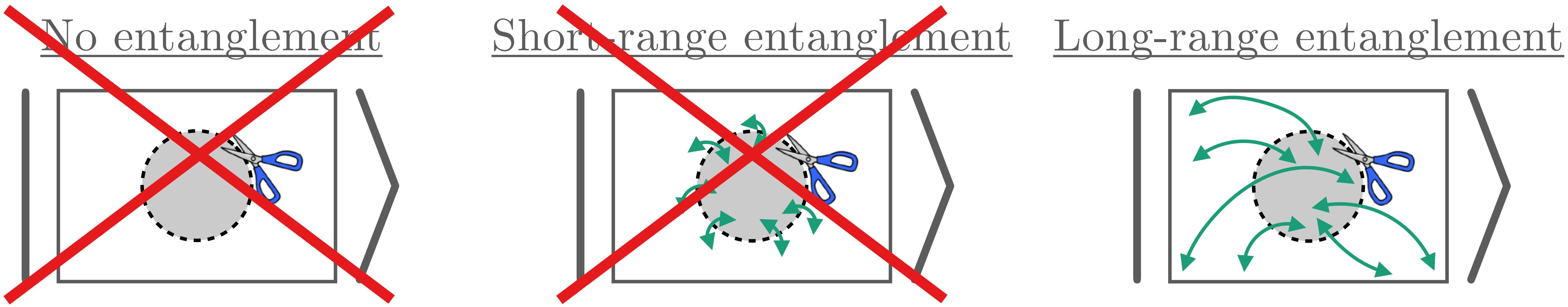
Symmetry can enforce states to have nontrivial entanglement



Example: The LSM theorem, SO(3) spin rotation and lattice translation symmetry

Symmetry-Enforced Entanglement

Symmetry can enforce states to have nontrivial entanglement



Example: The LSM theorem, SO(3) spin rotation and lattice translation symmetry

Spin	Trivially gapped phases?	Symmetry-enforced entanglement?
Integer	Exist	No
Half-integer	None	Yes

Second Lesson

Symmetry in quantum systems can constrain the ground state entanglement pattern

Generalizing Symmetry

Generalized Symmetries

The last ten years has seen tremendous progress in generalizing the notion of symmetry

Generalized Symmetries

Very quickly developing topic! At least 16 reviews:

1. McGreevy, Ann. Rev. Condensed Matter Phys. '22.
2. Córdova, Dumitrescu, Intriligator, Shao, Snowmass '22
3. Freed, Proc. Symp. Pure Math. '22
4. Reece, TASI '23
5. Shao, TASI '23
6. Brennan, Hong, '23
7. Bhardwaj et al., Phys. Rept. '23
8. Gomes, SciPost '23
9. Schäfer-Nameki, ICTP lectures, '23
10. Luo, QR Wang, YN Wang, Phys. Rept., '23
11. Carqueville, Del Zotto, Runkel, Encyclopedia of Math. Phys. '23
12. Iqbal, '24
13. Costa et al., Simons lectures '24
14. Davighi, '25
15. Kaidi, '26
16. **SP**, Schäfer-Nameki, Zhang, Ann. Rev. Condensed Matter Phys. (in progress)

Generalized Symmetries

The last ten years has seen tremendous progress in generalizing the notion of symmetry

➤ Crucially: Consequences of generalized symmetries are similar to those of ordinary symmetries

Generalized Symmetries

The last ten years has seen tremendous progress in generalizing the notion of symmetry

- Crucially: Consequences of generalized symmetries are similar to those of ordinary symmetries

They pass the duck test!

If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

Generalized Symmetries

The last ten years has seen tremendous progress in generalizing the notion of symmetry

➤ Crucially: Consequences of generalized symmetries are similar to those of ordinary symmetries

They pass the duck test!

If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

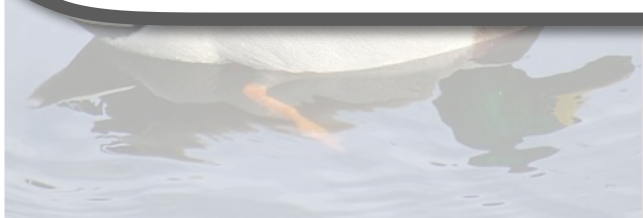
Generalized symmetries can also constrain low-energy dynamics and entanglement patterns

Generalized Symmetries

The last ten years has seen tremendous progress in generalizing the notion of symmetry

I want to introduce you to two generalized symmetries (there are many more)

1. Modulated symmetries
2. Non-invertible symmetries

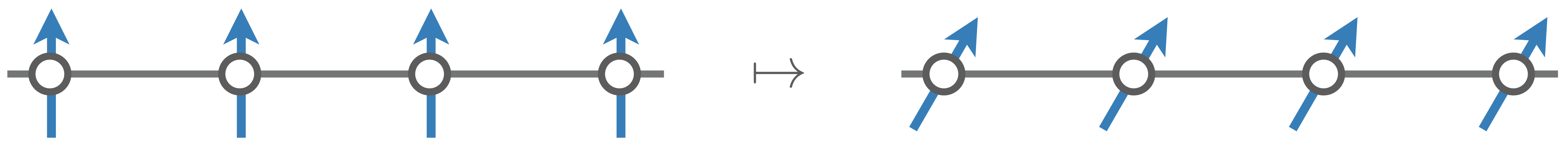


then it probably is a duck.

Generalized symmetries can also constrain low-energy dynamics and entanglement patterns

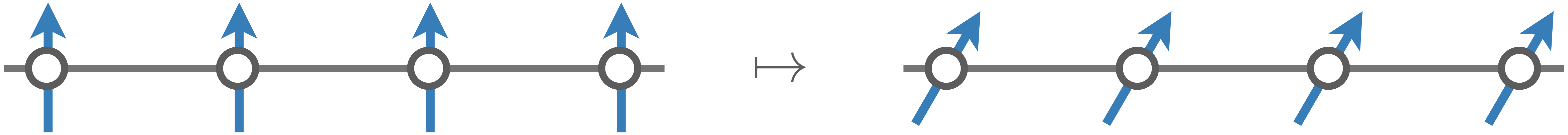
Modulated Symmetry

Ordinary symmetries act uniformly throughout space

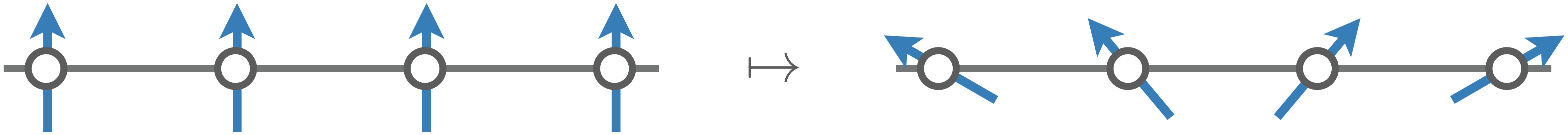


Modulated Symmetry

Ordinary symmetries act uniformly throughout space



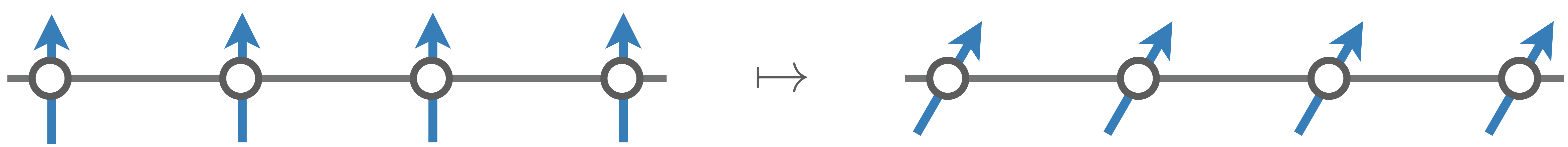
Modulated symmetries act in a spatially modulated way



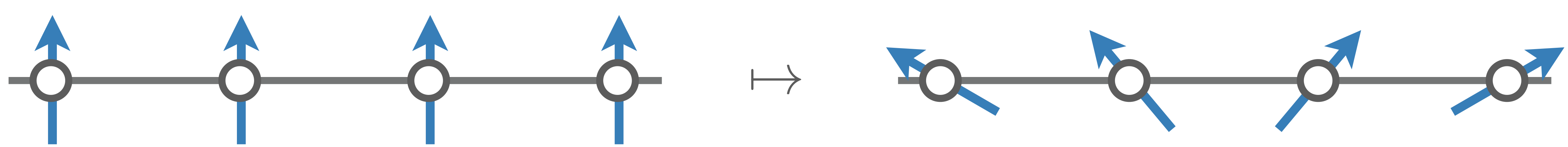
➤ Ubiquitous in quantum spin models and Lifshitz field theories

Modulated Symmetry

Ordinary symmetries act uniformly throughout space



Modulated symmetries act in a spatially modulated way



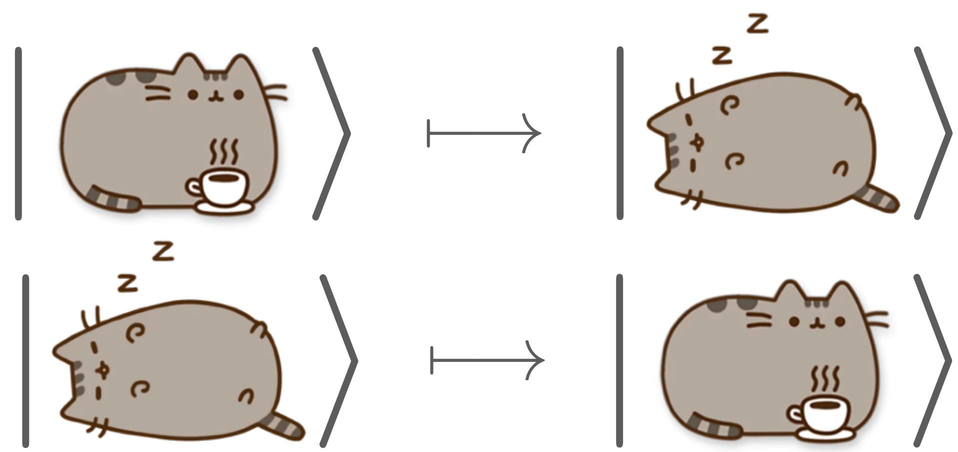
➤ Ubiquitous in quantum spin models and Lifshitz field theories

Modulated symmetries can constrain low-energy dynamics and entanglement patterns

[**SP**-Bulmash, arXiv '26]

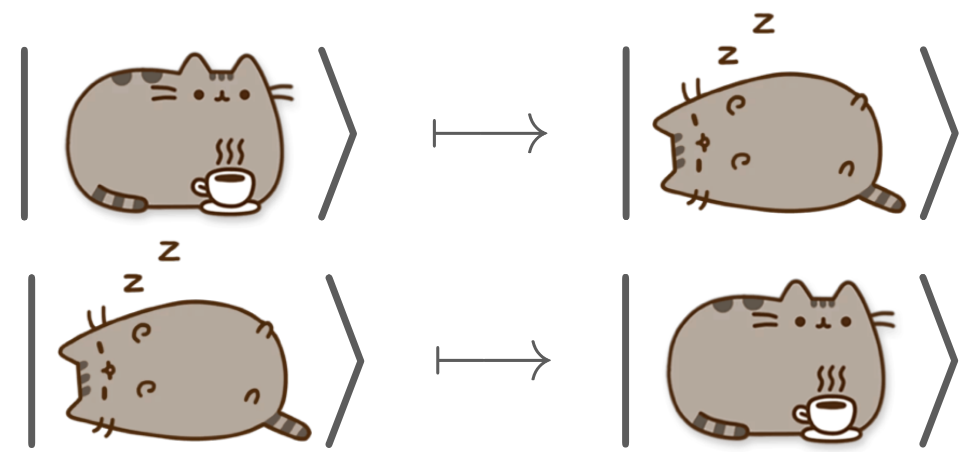
Non-Invertible Symmetry

Ordinary symmetry transformations have inverses

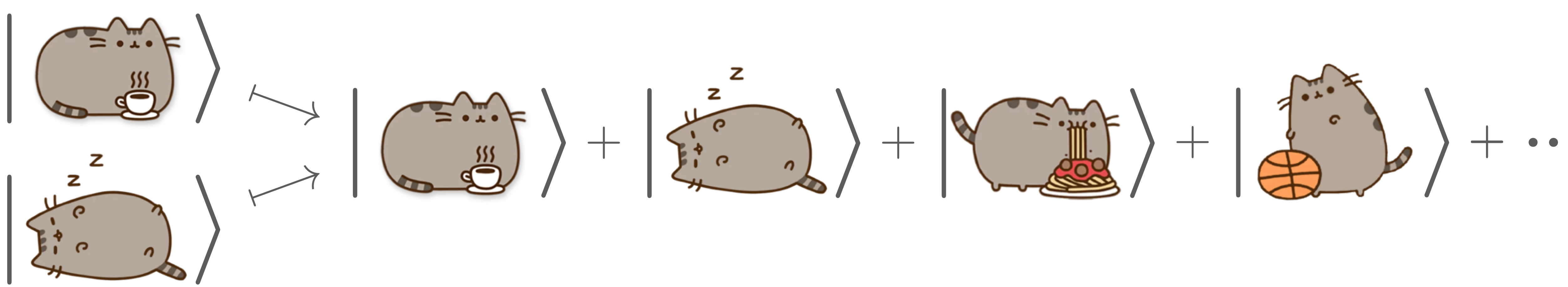


Non-Invertible Symmetry

Ordinary symmetry transformations have inverses



Non-invertible symmetry transformations have no inverse!



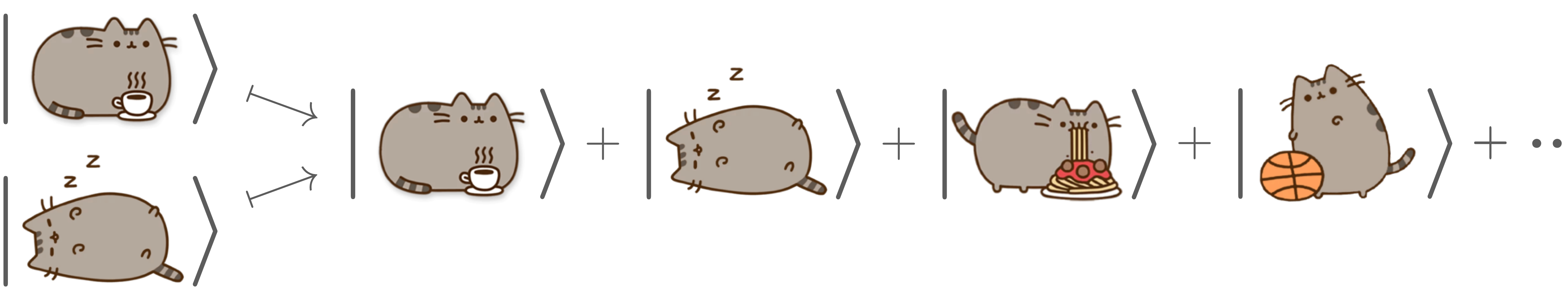
- Arise in simple quantum lattice systems and field theories

Non-Invertible Symmetry

Non-invertible symmetries can constrain low-energy dynamics and entanglement patterns


[**SP**-Lam-Aksoy, SciPost Phys. '25]
[**SP**-Aksoy-Lam, SciPost Phys. '26]

Non-invertible symmetry transformations have no inverse!



➤ Arise in simple quantum lattice systems and field theories

Third Lesson

Generalized symmetries also have powerful, unexpected consequences!

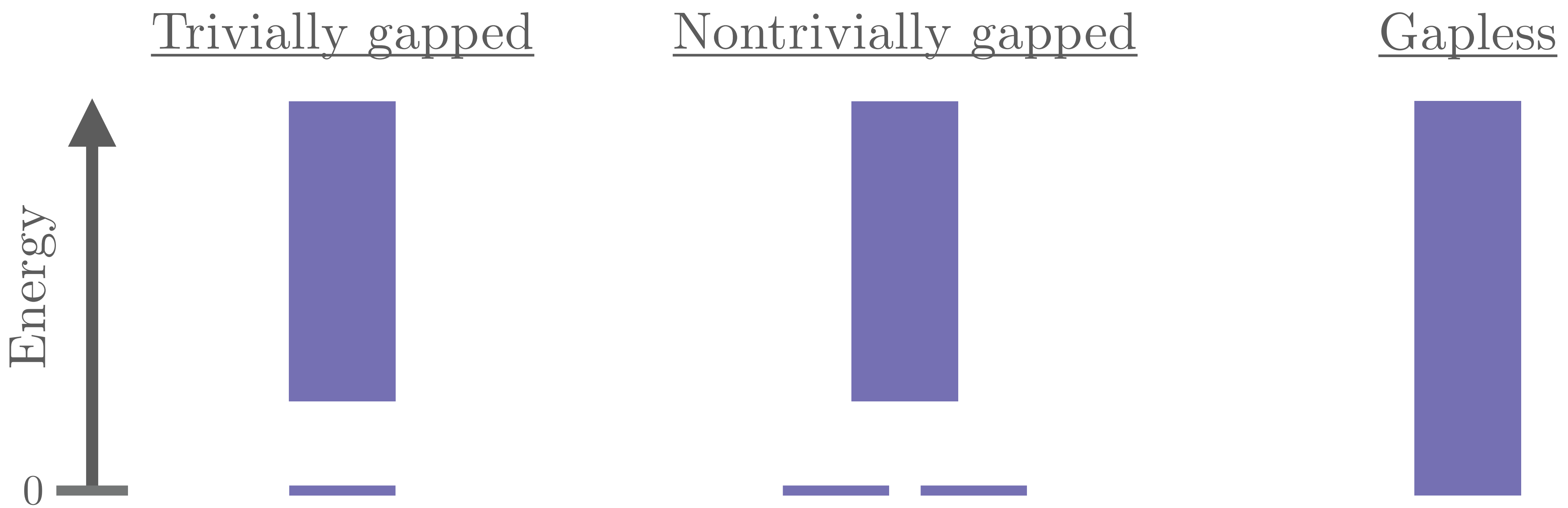
Summary

Symmetry in quantum physics has unexpected consequences

Summary

Symmetry in quantum physics has unexpected consequences

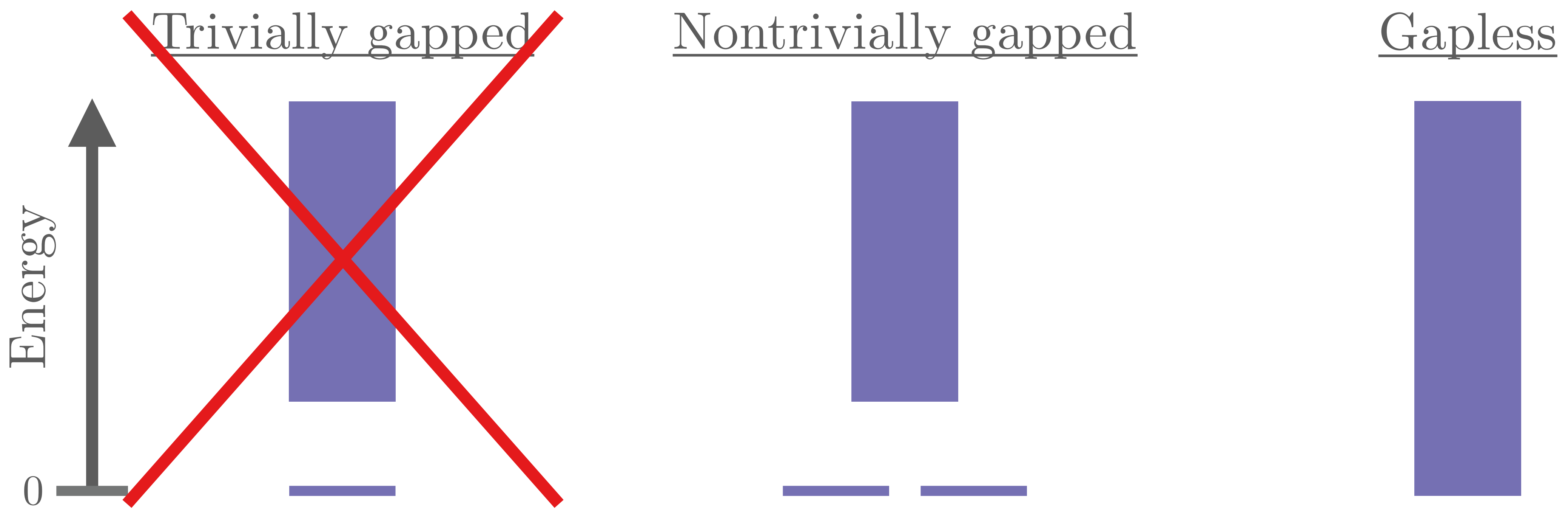
1. Constrains low-energy dynamics



Summary

Symmetry in quantum physics has unexpected consequences

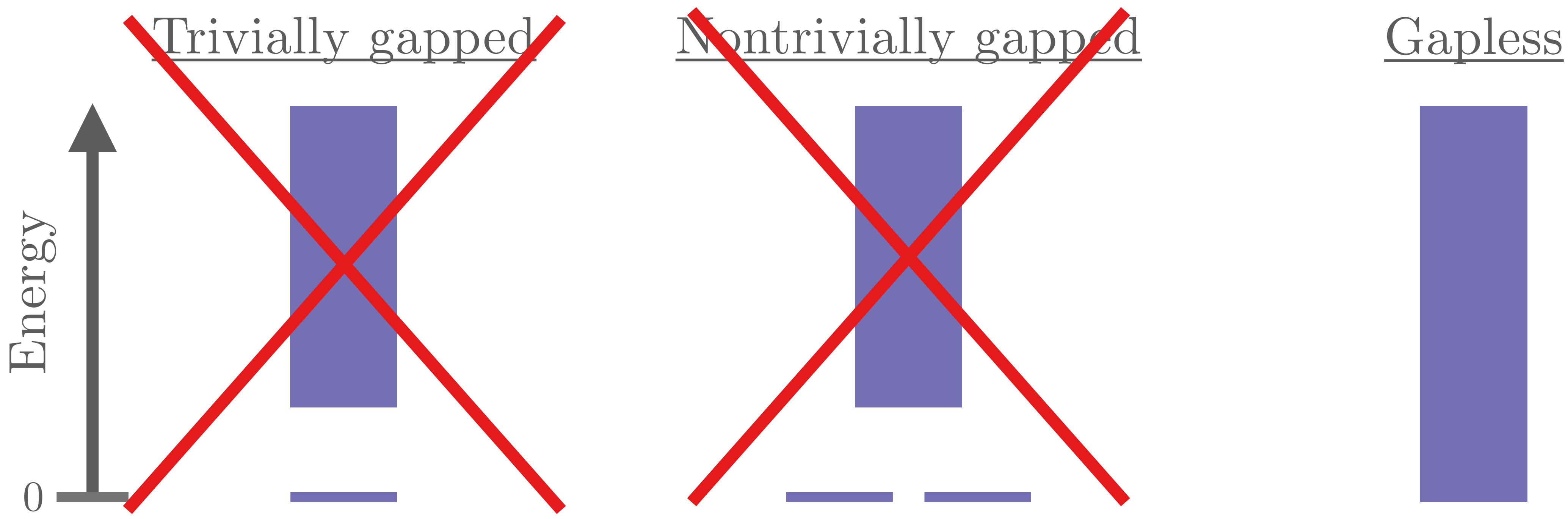
1. Constrains low-energy dynamics



Summary

Symmetry in quantum physics has unexpected consequences

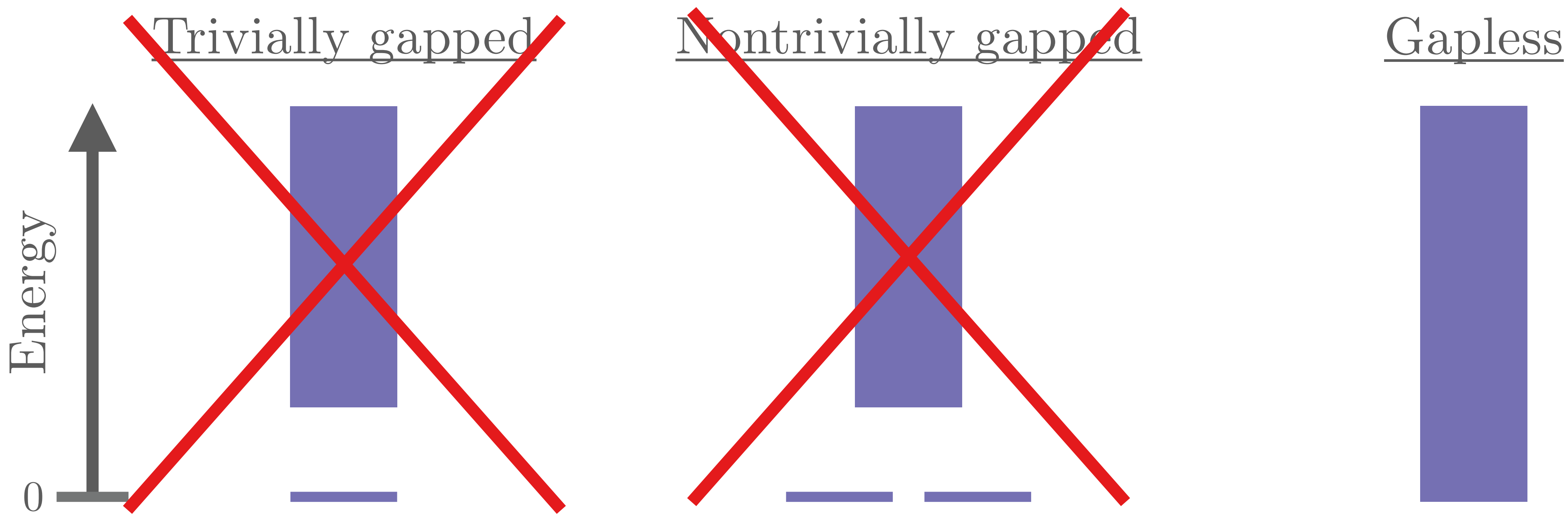
1. Constrains low-energy dynamics



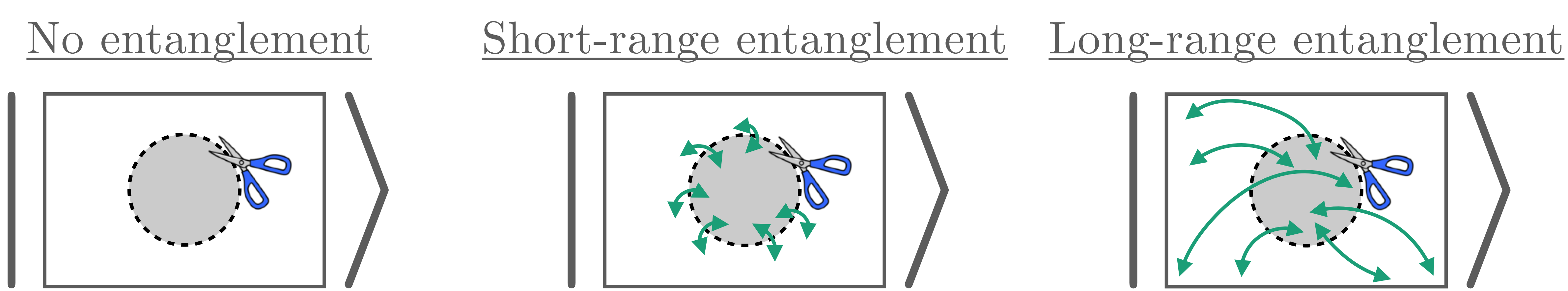
Summary

Symmetry in quantum physics has unexpected consequences

1. Constrains low-energy dynamics



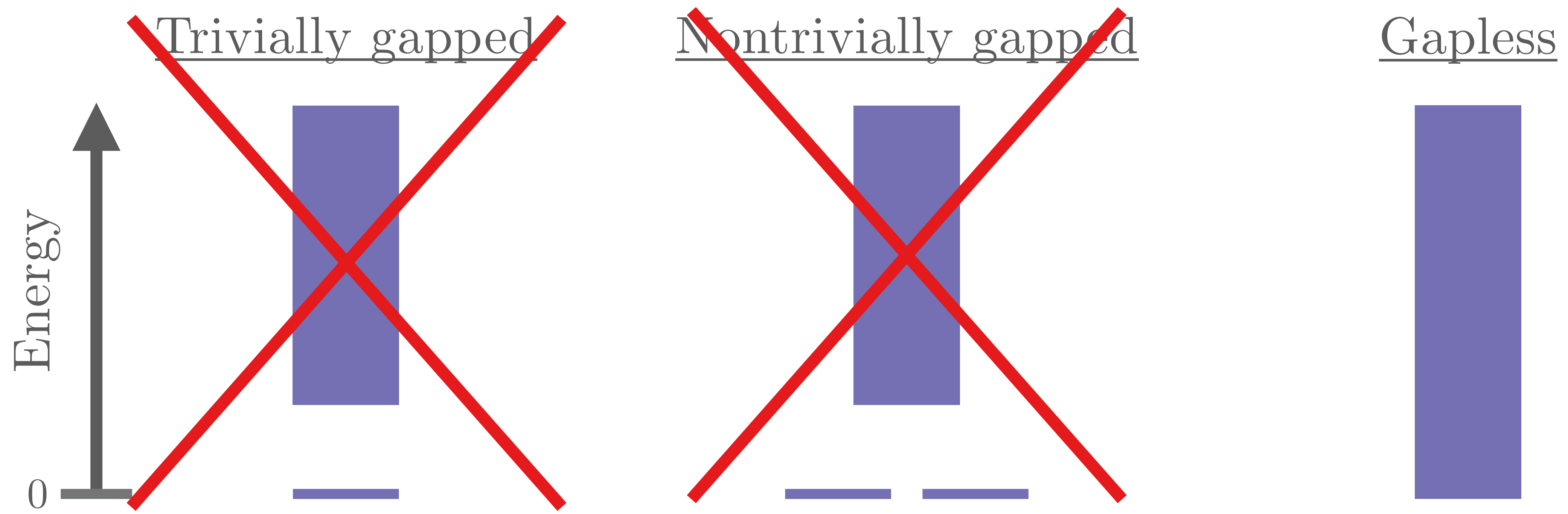
2. Constrains ground state entanglement



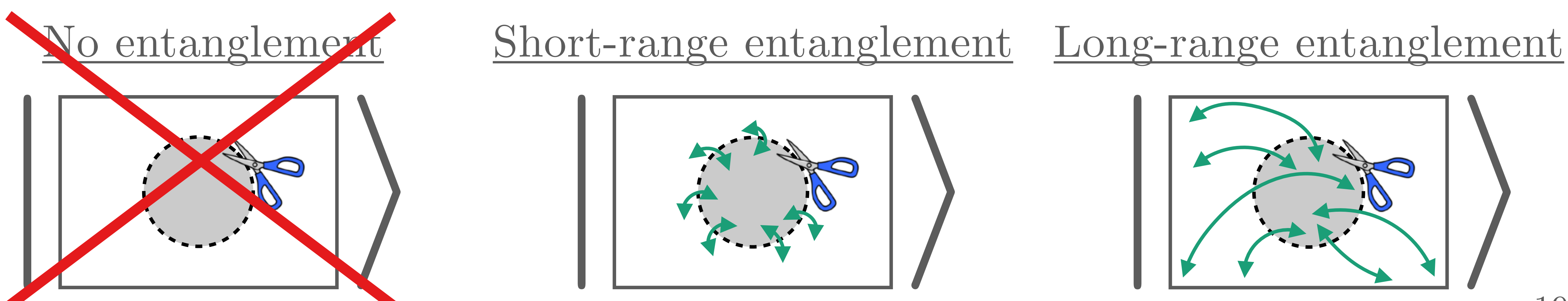
Summary

Symmetry in quantum physics has unexpected consequences

1. Constrains low-energy dynamics



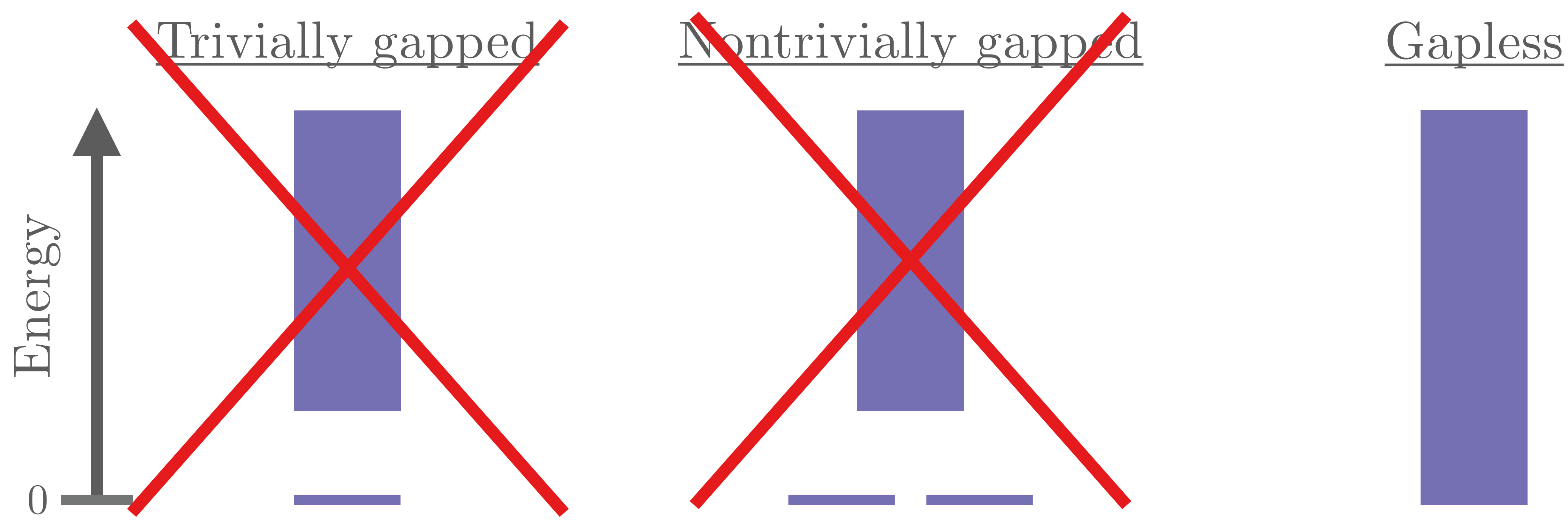
2. Constrains ground state entanglement



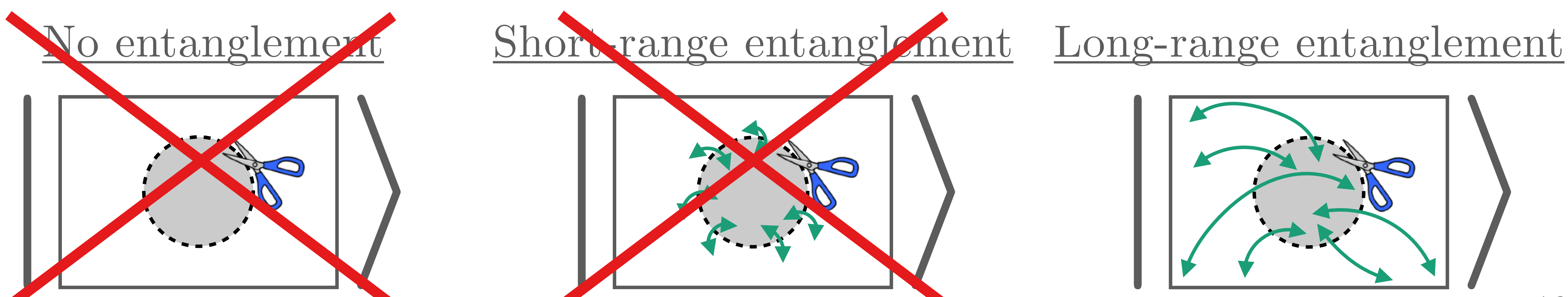
Summary

Symmetry in quantum physics has unexpected consequences

1. Constrains low-energy dynamics

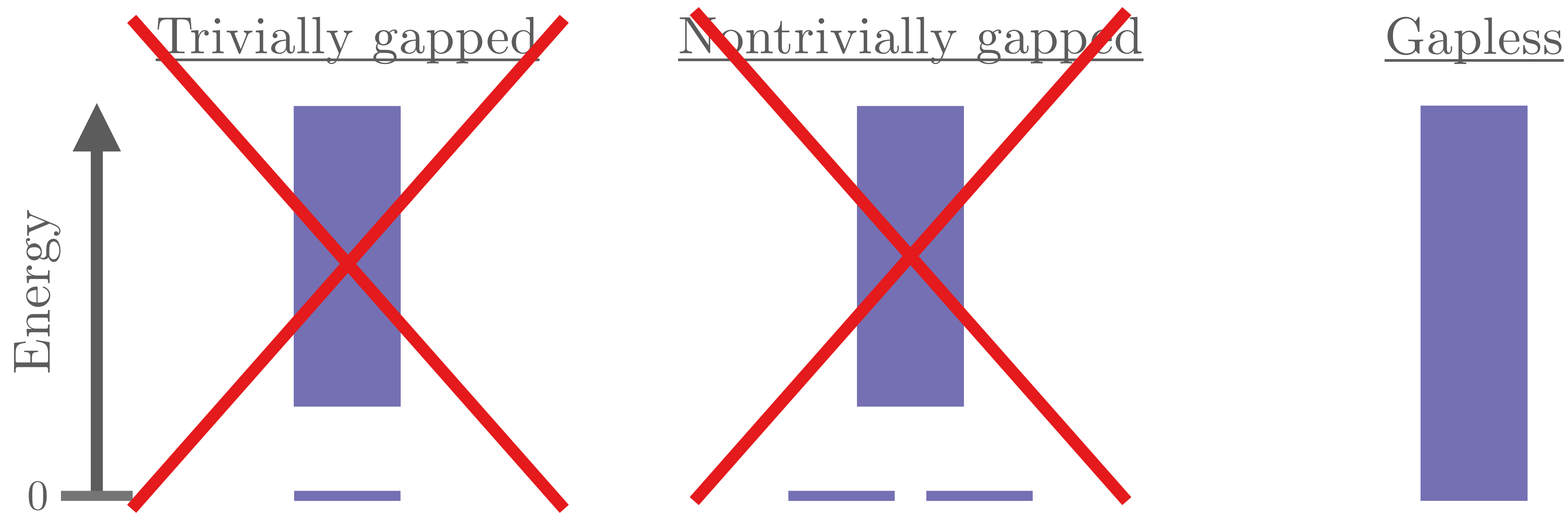


2. Constrains ground state entanglement

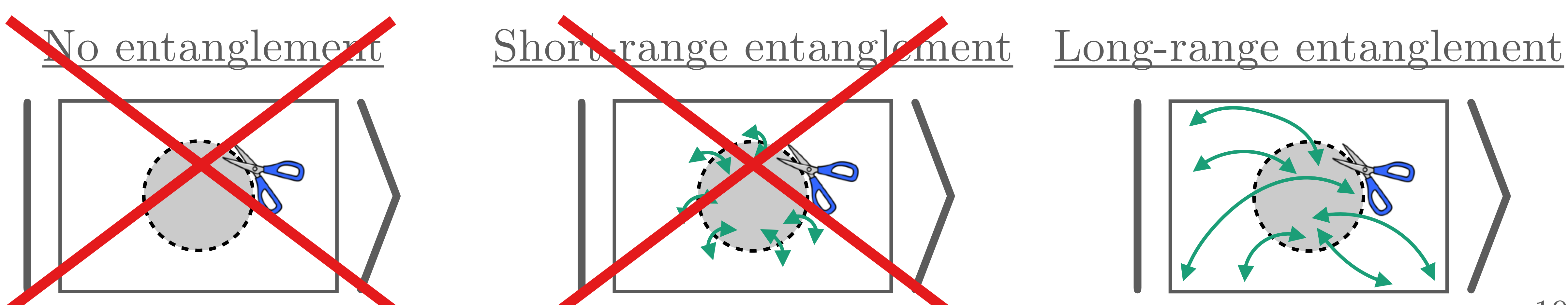


True for ordinary and generalized symmetries!

1. Constrains low-energy dynamics



2. Constrains ground state entanglement



Conclusion

Symmetry is both a foundation and frontier of physics

Conclusion

Symmetry is both a foundation and frontier of physics

- Amazingly interdisciplinary frontier!

Condensed Matter

Generalized Landau paradigm, New phases of matter, New LSM theorems, Anyons & fractons, Quantum disordering

High Energy

Confinement, Anomaly matching, Phenomenology, Geometric engineering, Amplitudes, Quantum gravity

Symmetry

Quantum Information

Sequential circuits, Fault-tolerant gates, Quantum cellular automata, Matrix product operators, ZX-calculus

Mathematical Physics

Category theory, Hopf algebra, Operator algebra, Cobordism, Stable homotopy, Homological algebra

Advertisement: Symmetry Seminar

Weekly virtual seminar series

sites.google.com/view/symmetryseminar

Condensed Matter

Generalized Landau paradigm, New phases of matter, New LSM theorems, Anyons & fractons, Quantum disordering

High Energy

Confinement, Anomaly matching, Phenomenology, Geometric engineering, Amplitudes, Quantum gravity

Symmetry

Quantum Information

Sequential circuits, Fault-tolerant gates, Quantum cellular automata, Matrix product operators, ZX-calculus

Mathematical Physics

Category theory, Hopf algebra, Operator algebra, Cobordism, Stable homotopy, Homological algebra