Intro to Lieb-Schultz-Mattis (LSM) and SPT-LSM theorems.

Main talk: An SPT-LSM theorem for weak SPTs with non-inv. sym.

→ Sal Pace, Ho Tat Lam, Ömer Aksoy arXiv: 2409.18113 [SciPost]

This Pre-talk

- all background for main talk
- all via examples

Outline:

1) Basics of SPTs and Symmetry defects.
2) LSM anomalies
3) SPT-LSM theorems

SPTs 101

SPT phase: gapped quantum phase protected by a symmetry with a unique gapped ground state (on all spatial manifolds)

- Interesting physics @ boundaries / Interfaces
- Characterized by response to static probes

e.g.) Background gauge fields

3+1D ordinary insulator: $\mathcal{L}[A] = \frac{1}{2}(E^2 - B^2)$

3+1D topological insulator: $\mathcal{L}[A] = \frac{1}{2}(E^2 - B^2) + \frac{\pi}{4\pi^2} E\cdot B$.

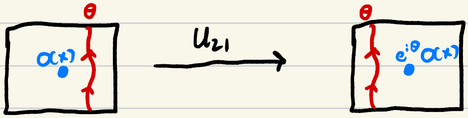
e.g.) Symmetry defects: localized modification to theory

$$H(\Sigma) = H + \delta H(\Sigma)$$
$$Q(\Sigma) = Q + \delta Q(\Sigma)$$
$$\vdots$$

→ Moved via unitaries (are topological defects)

$$H(\Sigma_2) = U_{21}\, H(\Sigma_1)\, U_{21}^{\dagger}$$

→ Implement sym across space



→ Twisted boundary conditions

$$(T_\perp)^L = \text{Sym op.}$$

Example: $\mathbb{Z}_2 \times \mathbb{Z}_2$ SPTs

- 1d closed chain w/ 2 qubits per site

First	Second
$H_p = -\sum_{j=1}^{L} (X_j + \tilde{X}_j)$	$H_c = -\sum_{j=1}^{L} \big(\underbrace{\tilde{Z}_{j-1} X_j \tilde{Z}_j}_{} + \underbrace{Z_j \tilde{X}_j Z_{j+1}}_{} \big)$ (Commute.)
$\lvert gs \rangle = \lvert + + \cdots + \rangle$	$\lvert gs \rangle = (\quad) \lvert gs \rangle = (\quad) \lvert gs \rangle$

- There is a $\mathbb{Z}_2 \times \tilde{\mathbb{Z}}_2$ sym

$$U = \prod_{j=1}^{L} X_j \qquad \tilde{U} = \prod_{j=1}^{L} \tilde{X}_j$$

- Both models have unique sym. gapped gs

H_p and H_c are in $\mathbb{Z}_2 \times \tilde{\mathbb{Z}}_2$ SPT phases

- H_p and H_c are different SPTs
  ⇝ check by inserting U sym defect : gives rise to twisted boundary conditions

$$Z_{j+L} = -Z_j$$

1) H_p unaffected :

$$U|gs\rangle = |gs\rangle \qquad \tilde{U}|gs\rangle = |gs\rangle$$

2) H_c becomes $\quad H_c + 2 Z_L \tilde{X}_L Z_1$

$$U|gs\rangle = |gs\rangle \qquad \tilde{U}|gs\rangle = -|gs\rangle$$

⇝ Different responses ⇒ different SPTs

$$Z_p[A,\tilde{A}] = 1 \qquad Z_c[A,\tilde{A}] = (-1)^{\int A \cup \tilde{A}}$$

LSM anomalies

A sym has an anomaly if it does not admit an SPT phase.

- Related to obstruction to gauging
- called an 't Hooft anomaly for internal symmetries
- called an LSM anomaly for internal + spatial syms.

Example

- 1d closed chain w/ 1 qubit per site

XY model: $$H = \sum_{j=1}^{L} \left(J_x X_j X_{j+1} + J_y Y_j Y_{j+1} \right) \qquad (Y = i Z X)$$

$\rightarrow$ Has lattice translations $T: O_j \rightarrow O_{j+1}$

$\rightarrow$ Has $\mathbb{Z}_2^x \times \mathbb{Z}_2^y$ symmetry

$$U_X = \prod_{j=1}^{L} X_j \qquad U_Y = \prod_{j=1}^{L} Y_j$$

U_X and U_Y furnish a local projective rep

$$U_X U_Y = (-1)^L U_Y U_X .$$

$\rightarrow$ Manifestation of an LSM anomaly between translations and $\mathbb{Z}_2^x \times \mathbb{Z}_2^y$

Heuristic

1) changing $L \rightarrow L+1$ is like inserting a translation defect

$$T^L = 1 \xrightarrow{\text{insert}} T^L = T$$

$\Rightarrow \mathbb{Z}_2^x \times \mathbb{Z}_2^y$ projectively represented in translation defect $\mathcal{H}$.
Like a type III anomaly.

2) Insert U_X sym. defect

$$H \rightarrow \tilde{H} = H + 2 Y_L Y_1$$

$$T \rightarrow \tilde{T} = X_1 T$$

$\rightarrow$ Now have projective algebra

$$\tilde{T} U_Y = - U_Y \tilde{T}$$

→ all states are doubly-degenerate

→ Implies H cannot have a unique gapped g.s.

Proof (See Ogata & Tasaki '19 for rigorous proof)

- Suppose H has unique gapped g.s. $|gs\rangle$
- In $L \to \infty$, $U_x(a,b) = \prod_{j=a}^{\infty} X_j$ acts on $|gs\rangle$ by

↳ localized near $j=a$.

$$U_x(a)\,|gs\rangle = \mathcal{U}_a\,|gs\rangle$$

→ Follows from split property

- Applies to area law states, which includes all 1+1D gapped gs.
- Says $\mathcal{H}$ factorizes as $L \to \infty$.

- $\mathcal{U}_a|gs\rangle$ is gs w/ single U_x defect inserted

→ Is the would be gs of $\tilde{H}$.
→ contradiction since $\tilde{H}$ has gsd $= 2$.

∴ U_x, U_y, and translations have an LSM anomaly

→ Consequences apply to any Symmetric Hamiltonian

SPT-LSM theorems

Bosonic SPTs in $(d+1)$D protected by anomaly-free G

Symmetry classified by

$$H^{d+1}(BG, U(1)) - \underline{\text{torsor}}$$

→ There is no canonical trivial SPT

→ Often times, for $\mathcal{H} = \otimes_j \mathcal{H}_j$, Product state is an SPT and Serves as a non-canonical trivial SPT.

→ SPT-LSM : Obstruction to this trivial SPT ⇒ all SPT States are entangled

<u>Example</u> [Jiang, Cheng, Qi, Lu '21]

1+1D system of $\mathbb{Z}_4 \times \tilde{\mathbb{Z}}_4$ qudits on $L \in 2\mathbb{Z}$ sites j.

→ $\mathbb{Z}_4$ qudit is 4-level system w/ clock and shift ops.

$$Z^4 = X^4 = 1 \qquad ZX = i XZ$$

→ Consider $\mathbb{Z}_4 \times \mathbb{Z}_4$ Symmetry operators

$$U = \prod_j X_j \tilde{X}_j \qquad V = \prod_j (Z_j \tilde{Z}_j)^{2j+1}$$

→ Local projective algebra

$$U_j V_j = - V_j U_j \qquad (UV = VU)$$

LSM anomaly?

→ Insert a U symmetry defect

$$T \longrightarrow \tilde{T} = U_1 T \qquad V \longrightarrow V$$

→ Now

$$T_{tw} V = W V T_{tw} \quad w/ \quad W = -\prod_j Z_j^2 \tilde{Z}_j^2$$

No LSM anomaly!

→ Not a projective algebra, instead a non-abelian group

→ Compatible w/ unique gapped gs if $W|gs\rangle = |gs\rangle$.

Projectivity still has consequenses ⇒ SPT - LSM theorem

→ Assume SPT ground state $|\psi\rangle = \bigotimes_j |\psi_j\rangle$

$$U|\psi\rangle = |\psi\rangle \qquad V|\psi\rangle = |\psi\rangle$$

→ bc U and V are onsite:

$$U_j|\psi\rangle = |\psi\rangle \qquad V_j|\psi\rangle = |\psi\rangle$$

→ But U_j and V_j anticommute ⇒ contradiction to SPT state req.

∴ $\bigotimes_j |\psi_j\rangle$ cannot be an SPT state.

Outlook

- Introduced: SPTs, LSM anomalies, and SPT-LSM theorem
- In Main talk: Add Non-invertible Symmetries!