

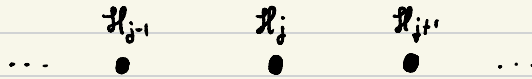
## Comments on Clifford QCA Symmetries

(Work in progress with Shu-Hong Shao)

Consider a 1+d quantum lattice system:

→ QM system  $\mathcal{H}_j$  @ site  $j$  with  $\dim \mathcal{H}_j = \text{finite}$ .

→ Total system: (Forms spatial lattice)



Hilbert space of finite region  $\Gamma$  is  $\bigotimes_{j \in \Gamma} \mathcal{H}_j$

eg) qubit system:  $\mathcal{H}_j = \mathbb{C}^2$

⇒ onsite operator algebra:  $A_j = M_2(\mathbb{C}) = \text{Span}_{\mathbb{C}} \{1, X, Y, Z\}$

## What is a symmetry operator? (Nat's talk)

Strictly locality-preserving Unitary Symmetries  $\Rightarrow$  a QCA  $\alpha$

→  $\alpha$  is a  $*$ -automorphism of the operator algebra st  $\alpha(O_j)$  has support on  $\{j-R, j-R+1, \dots, j+R\}$  ( $R = \text{range of } \alpha$ )

→ Finite lattice  $\Rightarrow \alpha(O_j) = U_\alpha O_j U_\alpha^\dagger$  For unitary  $U_\alpha$ .

Examples in qubit systems (assume finite lattice)

$U_\alpha$	QCA?
$\prod_j X_j$	Yes

onsite FQOC

nononsite FDQC

$$\prod_j \left( \frac{1}{2} (1 + Z_j + Z_{j+1} - Z_j Z_{j+1}) \right)$$

Yes

nononsite, non-FDQC

Lattice translations

Yes

$$e^{i\pm H}$$

No

(Finite time evolution)

(locality preserving with tails)

**Question:** How much of the QCA landscape is compatible with our usual perspectives of symmetry eg) gauging, anomalies, onsiteability

Consider simple scenario:

1) qubit system with clifford QCA Sym ops

→ a QCA that maps Pauli strings ( $i \prod_j X_j^{a_i} Z_j^{b_i}$ ) to Pauli strings.

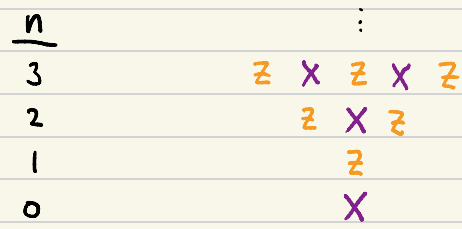
→ Interesting case.

2) cyclic sym group  $G = \langle \alpha \rangle = \mathbb{Z}_n$  or  $\mathbb{Z}$

**Example:**  $\alpha(X_j) = Z_j$        $\alpha(Z_j) = Z_{j-1} X_j Z_{j+1}$

→ Unitary  $U_\alpha = \prod_j CZ_{j,j+1} \prod_j H_j \Rightarrow$  clearly a FDQC.  
 ↳ Hadamard gate  $\frac{1}{\sqrt{2}}(X_j + Z_j)$

→ Operator spreading under  $\alpha^n$  (ignoring overall phase)



→ Won't show proofs. It all matches intuition.

→ Can prove the following:

$$1) \quad \langle \alpha \rangle = \begin{cases} \mathbb{Z} & \text{infinite lattice,} \\ \mathbb{Z}_L & \text{even } L \text{ sites,} \\ \mathbb{Z}_{4L} & \text{odd } L \text{ sites.} \end{cases}$$

2) There are no local symmetric operators. (besides  $\mathbb{1}$ )

→ like lattice translations, but  $\alpha$  is a FDQC.

3) There are gliders (local operator  $O_j$  st.  $\alpha(O_j) = O_{j+v}$ )

glider	$v$
$R_j = X_j Z_{j+1}$	1
$L_j = Z_{j-1} X_j$	-1

→ all gliders constructed from  $L_j, R_j$ .

4) Each symmetric local Hamiltonian takes the form

$$H_L(\{L_j\}) + H_R(\{R_j\})$$

with  $H_{L,R}$  translation-invariant

$$\rightarrow \text{Example: } H = \sum_j L_j + \sum_j R_j = \sum_j (Z_j X_{j+1} + X_j Z_{j+1})$$

$$\Rightarrow \text{Unitarily equivalent to } \sum_j (X_j X_{j+1} + Y_j Y_{j+1})$$

$$\Rightarrow \text{IR is } R = \sqrt{2} \text{ compact boson. } \langle \alpha \rangle \text{ becomes } \mathbb{Z}_2^c$$

→ A reason to care!

5) No symmetric gapped Hamiltonian with unique ground state

Interesting:  
No anomalous  
1+1d  $\mathbb{Z}$  Sym.

→ Every symmetric  $H$  also commutes with  $Y = \prod_j Y_j$ ,  
and  $\langle Y \rangle \times \langle \alpha \rangle = \mathbb{Z}_2 \times \mathbb{Z}$  has an anomaly.

Expectation: There is an SPT Hamiltonian after tensoring in ancillas. May require  $\infty$ -dim ancillas.

6)  $\alpha$  is not onsiteable bc gliders

7) Conjecture:  $\langle \alpha \rangle$  cannot be gauged

→ Intuition: no local symmetric operators  
(More detail: obstruction to find commuting gauss operators)

## Generalizations

There are 3 types of translation-invariant cQCA's  
(Gütschow et al 0906.3195)

1) Periodic cQCA  $\Rightarrow \alpha^n = 1$ .

2) glider cQCA  $\Rightarrow \alpha$  has gliders

3) fractal cQCA  $\Rightarrow \alpha^n$  transformation is diffusive.

(generic transl-inv cQCA is a fractal cQCA)

→ Example:  $\alpha(X_j) = Z_j$        $\alpha(Z_j) = Z_{j-1} Y_j Z_{j+1}$

$$U_\alpha = \prod_j C_{Z_{j,j+1}} \prod_j S_j H_j \quad S_j = e^{i\frac{\pi}{4}(1-Z_j)}, \quad \begin{array}{l} X_j \mapsto Y_j \\ Z_j \mapsto Z_j \end{array}$$

$\langle \alpha \rangle = \mathbb{Z}$  and  $H = \text{const.}$  is only sym Hamiltonian.

	periodic	glider	Fractal
Sym group on inf. lat.	$\mathbb{Z}_N$	$\mathbb{Z}$	$\mathbb{Z}$
Symmetric local ops.?	Yes	Sometimes	No
Gauge-able?	Sometimes	Sometimes?	No?
Onsiteable?	Sometimes	No	No
$\exists$ nontrivial sym Ham?	Yes	Yes	No
$\exists$ SPT Hamiltonian?	Sometimes	Sometimes	No