LSM anomalies of modulated symmetries

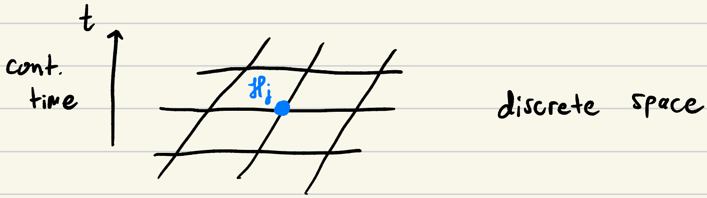
To appear tonight, w Danny Bulmash

Fundamental part of a quantum theory's DNA

Anomalies are powerful (constrain phases, dualities, inflow)

— Exist on lattice and in QFT, with or without fermions

Here: Anomalies of symmetries in lattice systems



lattice anomaly knowledge << QFT anomaly knowledge

(def of sym + anom on lat?)

(lat. and cont. form symbiotic relation.)

lattice sys $\xleftrightarrow{\text{Knowledge}}$ qft

Morally like 't Hooft anom

Lieb-Schultz-Mattis (LSM) anomalies := obstructions to trivial symmetry-preserving gapped phases with crystalline symmetries

Ömer's talk

— original LSM anomaly: Spin 1/2 chain w $G = SO(3) \times \mathbb{Z}$ Sym.

spin rotation — lattice transl.

→ SO(3) sym op $\prod_j \underbrace{e^{i\frac{\theta}{2}\hat{n}\cdot\vec{\sigma}_j}}_{\text{proj } SO(3) \text{ rep.}}$

— Best understood for non-gen sym with group

$$G = G_{int} \times G_{cr}$$

internal sym group — crystalline sym group

Ho Tat's talk

- More generally

$$1 \to G_{int} \to G \to G_{cr} \to 1$$

When extension splits, $G = G_{int} \rtimes G_{cr}$

- G_{cr} action on G_{int}

$$\rho : G_{cr} \longrightarrow \mathrm{Aut}(G_{int})$$
$$g_{cr} \longmapsto \rho_{g_{cr}}$$

- when ρ is nontrivial $\Rightarrow$ Modulated Sym

example: dipole sym in 1+1D lattice system

- Charge operators: $Q = \sum_j q_j$ and $P = \sum_j j q_j$

$$TPT^\dagger = \sum_j j\, q_{j-1} = P + Q \Rightarrow (U(1) \times U(1)) \rtimes \mathbb{Z} \text{ sym.}$$

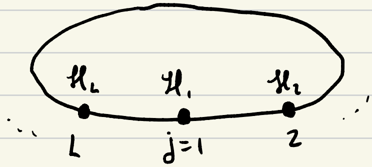
Modulated syms are interesting:

- SSB: goldstones in 1+1 D
- Kramers-Wannier sym: Non-inv Spatial reflections
- Gauge theory: Fractons and UV/IR mixing

This talk: LSM anomalies of modulated sym

- Restrict to bosonic systems in 1+1D with finite G_{int} and G_{cr} sym is lattice transl.

Set up: Spatial lattice of L sites on a ring



$$\mathcal{H} = \bigotimes_{j=1}^{L} \mathcal{H}_j$$

finite dim $\mathcal{H}_j \cong \mathcal{H}_{site}$

$G_{int} \rtimes \mathbb{Z}_L$ sym ops

acts on $\mathcal{H}_j$.

$$U_g = \prod_{j=1}^{L} U_g^{(j)} \qquad (g \in G_{int})$$

$$T^n \qquad (n \in \mathbb{Z}_L)$$

With

$$U_g^{(j)} U_h^{(j)} U_{gh}^{(j)\dagger} = \nu_j(g,h) \in U(1) \qquad (1)$$

$$T^L = 1$$

$$T U_g^{(j)} T^\dagger = U_{\rho_1(g)}^{(j-1)} \qquad (2)$$

- From (2), $T U_g T^\dagger = U_{\rho_1(g)}$ $\big(\rho : \mathbb{Z}_L \to Aut(G_{int})\big)$

onsite proj rep

LSM anomalies arise from $[\nu_j] \in \mathcal{H}^2(G_{int}, U(1))$

- conjugate (1) by T : $\nu_j(g,h) = \nu_{j-1}(\rho_1(g), \rho_1(h))$

$$\Rightarrow \qquad [\nu_j] = \rho_j^*[\nu] . \qquad \text{(translation covariance)}$$

When does $[\nu]$ lead to an LSM anomaly?

Consider

$$\mathcal{H}_j = \mathcal{V} \otimes \mathcal{W}, \qquad U_g = \prod_{j=1}^{L} V_g^{(j)} \otimes W_g^{(j)}, \qquad T = T_v T_w$$

proj reps $[\gamma_j]$ $\qquad$ $[\mu_j]$

- $G_{int} \times \mathbb{Z}_L$ sym: $T_{v/w}$ act as (2)

$$\Rightarrow \quad [\gamma_j] = \rho_j^*[\gamma] \qquad [\mu_j] = \rho_j^*[\mu]$$

- LSM anomaly-free: $[\nu_j] = [\gamma_j] + [\mu_j] = 0$

Rewrite $\quad U_g = \prod_{j=1}^{L} W_g^{(j)} \otimes V_g^{(j+1)}$

- Now $[\nu_j] = [\gamma_{j+1}] + [\mu_j] \Rightarrow [\nu] \simeq (\rho_1^* - 1)[\gamma]$

$\therefore$ $G_{int} \times \mathbb{Z}_L$ is LSM anomaly-free if $[\nu] \in \mathrm{Im}(\rho_1^* - 1)$

Can show: LSM anomaly free iff $[\nu] \in \mathrm{Im}(\rho_1^* - 1)$

ρ	$[\nu]$	LSM anomaly
trivial or nontrivial	$= 0$	No
trivial	$\neq 0$	Yes
nontrivial	$\neq 0$	Maybe

This should be suprising!

Classification : $H^2(G_{int}, U(1)) / \mathrm{Im}(\rho_1^* - 1)$

- For infinite lattice, $G = G_{int} \rtimes \mathbb{Z}$

Lyndon-Hochschild-Serre.

$$\frac{H^2(G_{int}, U(1))}{\mathrm{Im}(\rho_1^* - 1)} \cong H^1_{\rho^*}(\mathbb{Z}, H^2(G_{int}, U(1))) \subset H^3(G, U(1))$$

nontrivial $\mathbb{Z}$-module

Example : Exponential Sym

$G_{int} = \mathbb{Z}_N \times \mathbb{Z}_N$ w $(g_1, g_2) \in \mathbb{Z}_N \times \mathbb{Z}_N$ and additive group law.

$$\rho_n(g_1, g_2) = (a^n g_1, b^n g_2)$$

→ Integers a, b coprime to N

→ Can show $\rho_1^*[\nu] = ab[\nu] \quad \forall \quad [\nu] \in H^2(G_{int}, U(1)) = \mathbb{Z}_N$.

let $[\nu] = K[\alpha]$ with $[\alpha]$ a generator of $H^2(G_{int}, U(1))$
with 2-cocycle $\alpha(g,h) = \exp\left[\frac{2\pi}{N} i g_1 h_2\right]$

→ $[\nu] \in \mathrm{Im}(\rho_1^* - 1)$ iff $\gcd(ab-1, N) \mid K$.

↳ LSM anomaly-free cond

Case	LSM anomaly for $[\nu] \neq 0$?
$ab = 1 \mod N$	Always
$\gcd(ab-1, N) = 1$	Never
$\gcd(K, N) = 1$	iff $\gcd(ab-1, N) \neq 1$

Outlook

LSM anomalies of modulated sym w lattice transl in 1+1D

→ LSM anomaly existence depends on onsite proj rep and homomorphism ρ.

What else is in arXiv: 2602.XXXXX

1) Examples of 1+1D stabilizer code models

   - SPT-LSM theorems

2) LSM anomalies of $G_{int} \rtimes pm$

3) Beyond 1+1D

   - Stratified anomalies: generalization of onsite proj reps

4) Examples in 2+1D.