Symmetry-enforced Fermi Surfaces

Based on arXiv:25(next week) w Luke Kim and Shu-Heng Shao

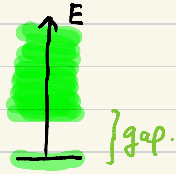
1) Anomalies and Sym. enforced gaplessness
2) Sym-enforced Fermi Surfaces
3) Generalized Onsager Sym.

Landscape of quantum phases

Phases of quantum many-body systems come in two flavors
(distinguished by many-body energy spectrum)

1) gapped phases:

- Finite energy-gap between GS and 1st excited state
  (In thermodynamic limit)

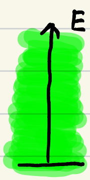


- well understood, rich variety:

  discrete SSB, SPT, Topological order, Fractons
  (Both TQFT and beyond TQFT gapped phases)

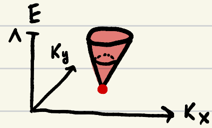
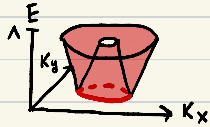
2) gapless phases:

- No gap between GS and excited states
  (In thermodynamic limit)



(much less understood than gapped counterparts)

- Not all gaplessness is equal

# gapless modes	Example	Spectrum
finite	goldstone bosons	
∞	Fermi surfaces (FS)	

Interesting to identify constraints that forbid gapped phases

Sym-enforced gaplessness

Recall: anomalous symmetries forbid trivial gapped phases

- Compatible with SSB, topological/fracton order, gaplessness

Some anomalies are even stronger: Sym-enforced gaplessness

- Compatible with SSB or gaplessness

$\Rightarrow$ guaranteed gapless if sym-preserved

- Examples:

1) Wang-Senthil $(SU(2) \times \mathbb{Z}_4^T)/\mathbb{Z}_2$ anomaly } Can have gapped SSB phase

   Wang, Senthil 2014

2) Witten's SU(2) anomaly } No allowed gapped phases

   García-Etxebarria et al 2017

Known examples compatible w/ finite and ∞ # gapless modes

- What about sym-enforced "∞-gaplessness"?

Sym-enforced FS?

Set up

(2+1)D Quantum lattice model w spatial $L_x \times L_y$ square lattice and cont. time.

(Warning: working all within timeslice of spacetime)

- Lattice vectors $r = n_x \hat{x} + n_y \hat{y}$ w $n_i \in \mathbb{Z}$ and $n_i \sim n_i + L_i$.
- One complex fermion per site: fermionic creation/annihilation ops

$$\{c_r^\dagger, c_{r'}\} = \delta_{r,r'} \qquad \{c_r, c_{r'}\} = 0$$

- Local Hamiltonian $H = \sum_r H_r$

Goal: find operators $\{U_g \mid g \in G\}$ s.t.

1) U_g is a unitary G sym op: $U_g U_h = U_{gh}$, $g,h \in G$.

2) $[U_g, H] = 0 \implies$ H <u>always</u> has a FS

   - Sufficient but not necessary cond. for a FS.

Symmetries

Symmetry must forbid all terms that can destroy a F.S.

e.g.) - Chemical potential term $-\mu \sum_r n_r$ $(n_r = c_r^\dagger c_r)$

- Pairing terms: $\sum_{\langle r,r' \rangle} (\Delta c_r c_{r'} + hc)$

- Density-density interactions: $\sum_{\langle r,r' \rangle} n_r n_{r'}$

An obvious sym: $U(1)$ w charge $Q = \sum_r (n_r - 1/2)$

↳ assume $L_x L_y$ even.

$$e^{i\theta Q} c_r e^{-i\theta Q} = e^{-i\theta} c_r$$

→ rules out pairing term

How to forbid the other terms?

Real/Majorana fermion operators

$$a_r = c_r^\dagger + c_r, \qquad b_r = i(c_r^\dagger - c_r).$$

$$a_r^\dagger = a_r \qquad b_r^\dagger = b_r \qquad \{a_r, b_{r'}\} = 0$$

$$\{a_r, a_{r'}\} = \{b_r, b_{r'}\} = 2\delta_{r,r'}$$

→ $n_r = \frac{i}{2} a_r b_r + \frac{1}{2}$ ⇒ Try to decouple a_r and b_r

Majorana translation sym

$$T_v^{(b)} \begin{pmatrix} a_r \\ b_r \end{pmatrix} T_v^{(b)\dagger} = \begin{pmatrix} a_r \\ b_{r+v} \end{pmatrix}$$

→ due to locality, causes a and b majoranas to decouple:

$$H = H(a) + H(b)$$

→ $T_v^{(b)} c_r T_v^{(b)\dagger} = \frac{1}{2}\left(c_r^\dagger + c_r - c_{r+v}^\dagger + c_{r+v}\right)$

Enforcing these sym requires:

H commutes w Q, $T_{\hat{x}}^{(b)}$, and $T_{\hat{y}}^{(b)}$

Can show most general sym local Hamiltonian is

$$H = \sum_{\substack{\text{all} \\ r}} \sum_{\substack{\text{finite} \\ v}} i g_v c_r^\dagger c_{r+v} \qquad (g_{-v} = -g_v \in \mathbb{R})$$

→ Sym enforced free fermions.

$c_K = \frac{1}{\sqrt{N_{sites}}} \sum_r e^{-iK\cdot r} c_r$

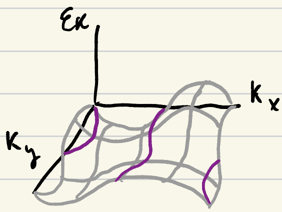
→ In momentum space $H = \sum_{K \in BZ} \varepsilon_K c_K^\dagger c_K$ where $\varepsilon_K = -2 \sum_v g_v \sin(K\cdot v)$

⇒ ground state : $c_K^\dagger c_K |gs\rangle = \delta_{\varepsilon_K < 0} |gs\rangle$

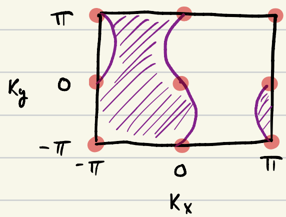
⇒ FS is a 1-dim locus where $\varepsilon_K = 0$.

e.g.) $H = -\frac{i}{2} \sum_r \left(c_r^\dagger c_{r+\hat{x}} + \frac{3}{4} c_r^\dagger c_{r+\hat{y}} \right) + h.c.$

$$\varepsilon_K = \sin(K_x) + 3/4 \sin(K_y)$$



@ $\varepsilon_K = 0$



Dispersion satisfies $\varepsilon_{-K} = -\varepsilon_K$:

1) H always has a FS (Follows from intermediate value theorem)

2) half-filling. ⇒ $Q |gs\rangle = 0$

3) Generic FS always topologically non-trivial

→ Each point $K \equiv -K$ lies on a non-contractible component of the FS

Sym-enforced FS from U(1) and Maj translations

UV Sym group Not U(1) x Maj transl.

Q and $T_v^{(b)}$ do not commute :

$$T_v^{(b)} Q T_v^{(b)\dagger} = \frac{i}{2} \sum_r a_r b_{r+v} \equiv Q_v$$

$$[Q_v, Q_{v'}] = i G_{v'-v} \quad \text{where} \quad G_v = \frac{i}{2} \sum_r (a_r a_{r+v} - b_r b_{r+v})$$

$$[G_v, G_{v'}] = 0$$

$$[Q_v, G_{v'}] = 2i (Q_{v-v'} - Q_{v+v'})$$

→ Span$\{Q_v, G_v\}$ forms $O(N_{sites})$ dim. Lie algebra

→ Has Onsager subalgebras, e.g., Span$\{Q_{n\hat{x}}, G_{n\hat{x}}\}$.
Vernier, O'Brien, Fendley 2018, Chatterjee, SP, Shao 2024

Full Sym group $\mathrm{Ons}_2 \rtimes (\mathbb{Z}_{L_x} \times \mathbb{Z}_{L_y})$
↳ generalized Onsager Sym

Includes transformation $c_K \mapsto e^{-if(K)} c_K$ w $f(K) = f(-K)$

→ Sym op $e^{i\sum_K f(K) Q_K}$ w $Q_K = \frac{2}{N_{sites}} \sum_v \cos(K\cdot v) Q_v$

→ Anomaly-free: commutes w $H = -\mu \sum_r n_r$
→ For $K \in F$, this is a subgroup of the $LU(1)$ Sym from Else, Thorngren, Senthil 2020

Summary

$U(1)$ + Maj translation = $\mathrm{Ons}_2 \rtimes$ transl ⇒ Fermi Surface

⤳ discussed square lattice, but applies to and d-dim Bravais lattice

Follow-up questions:

- Stability to ancillas?

- anomaly matching of $LU(1)$?
- what about codim-p Fermi surfaces?