# Infinite-Order Lattice anomalies from CPT.

't Hooft anomalies ⊂ QFT Structural data

- obstruction to gauging a global Symmetry: $Z[A] = Z[A+df]\, e^{i\theta(A,f)}$
- constrains IR dynamics (Not inconsistency of theory)

  ⇒ IR : SSB, TQFT, CFT, ... (Cannot flow to a trivial theory)

- Not a "yes or no" thing

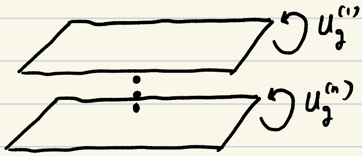
  ⇒ A Sym can be anomalous in different ways

An important characteristic

**Order of an anomaly**

Consider anomalous $G$ symmetry operators $\{U_g\}_{g\in G}$.

- Take $n$ copies of the system




- order of anomaly = Min $n$ s.t. $\{\bigotimes_{i=1}^{n} U_g^{(i)}\}_{g\in G}$ is anomaly-free.

  ( = order of anomaly inflow theory under stacking)

⇒ If $n$ exists, anomaly is finite-order (eg, finite symmetries)

otherwise, anomaly is infinite-order (eg, perturbative anomalies) eg) QM $\mathbb{Z}_N \times \mathbb{Z}_N$ Sym : $U^N = V^N = 1, \quad UV = e^{\frac{2\pi i}{N} K} VU$

- Different $K \in \{0, 1, \dots, N-1\}$ $\Rightarrow$ different anomaly.
- order of anomaly $= N / \gcd(N, K)$

## This talk

Anomalies in quantum lattice systems w finite dim dof.
(quantum lattice systems → spatial lattice, continuum time.)

- [Kapustin, Sopenko '24 & Liu '26] : No infinite-order anomalies for sym that (1) are locality-preserving and (2) w a compact Lie group

Evade no-go using (1) CPT and (2) non-compact Lie group
( w Elijah Lew-Smith and Shu-Heng Shao )

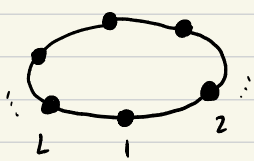
Why?
- Practical reason: Regularizing QFT
- Conceptual reason: Anomalies on lattice not well understood

Perspective for this talk
Lattice anomaly: obstruction to trivially gapped phase

(Utility: Avoids gauging. used for non-invertible Sym in QFT, too.)

## 1+1d Hamiltonian lattice system

Finite spatial lattice, 2 Majoranas $a_j$ and $b_j$ per site $j$.




$a_j^\dagger = a_j \qquad \{a_j, a_{j'}\} = 2\delta_{j,j'}$

Same for $b_j$

$\{a_j, b_{j'}\} = 0$

To ground ourselves

$$H = \frac{i}{2}\sum_{j=1}^{L}\left(a_j a_{j+1} + b_j b_{j+1}\right) \xrightarrow{\text{IR}}$$

$\mathcal{L} = i\psi_L^\dagger(\partial_t + \partial_x)\psi_L + i\psi_R^\dagger(\partial_t - \partial_x)\psi_R$

Free, massless Dirac Fermion

$\psi = (\psi_L \quad \psi_R)$

write order

( Chatterjee, SP, Shao PRL '25 )

$[Q_V, Q_A] \neq 0$

Forms Onsager algebra

$$Q_V = \frac{i}{2}\sum_j a_j b_j$$

$$Q_A = \frac{i}{2}\sum_j a_j b_{j+1}$$

$U(1)_V: \ \psi \to e^{-i\theta}\psi$

$U(1)_A: \ \psi \to e^{-i\theta\sigma^z}\psi$

Lattice anomaly: Symmetry enforced gaplessness

Infinite-order anomaly

$$S_{\text{inflow}} = \frac{i}{\pi}\int A_V dA_A \quad \begin{cases} A_L = A_V + A_A \\ A_R = A_V - A_A \end{cases}$$

Order of onsager sym anomaly? Two

- double system: 4 majoranas per site $a_j^{I}, b_j^{I}, a_j^{II}, b_j^{II}$
- diagonal sym gen by $Q_V = Q_V^{I} + Q_V^{II}$ and $Q_A = Q_A^{I} + Q_A^{II}$

$\Rightarrow$ Anomaly-Free: $H_{gap} = \frac{i}{2}\sum_j\left(a_j^{I} a_j^{II} + b_j^{I} b_j^{II}\right)$ has symmetry and a unique gapped ground state.

$\Rightarrow$ Not a contradiction...

$\Rightarrow$ IR anomaly is infinite-order, so natural to ask:

Can this lattice anomaly's order be changed?

Yes! enforce more symmetry

Consider anti-unitary $\Pi$ satisfying

$$\Pi \begin{pmatrix} a_j \\ b_j \end{pmatrix} \Pi^\dagger = \begin{pmatrix} a_{-j+1} \\ b_{-j+1} \end{pmatrix}$$

$\to$ generates an anti-unitary $\mathbb{Z}_2$ reflection symmetry.

$\Rightarrow$ Lattice CPT symmetry ( Flows to the CPT sym of Dirac QFT )

→ $\Pi$ in doubled System: $\Pi H_{gap} = - H_{gap} \Pi$
gets rid of term responsible for onsager Sym anomaly's finite order

Now enforce lattice Sym generated by $Q_V$, $Q_A$, and $\Pi$.

→ Can prove n copies of system is always gapless
( Lew-Smith, SP, Shao to appear )

Onsager + lattice CPT symmetry ⇒ infinite-order lattice anomaly.

**Recap**

| Lattice Sym | anomaly order | anomaly order w CPT |
|---|---|---|
| (This row first) Onsager algebra | 2 (Different from continuum) | $\infty$ (Matches continuum) |
| Wucheng's talk ⇝ Seiberg, Shao, Zhang '26 Majorana translations | 2 (Different from continuum) | 8 (Matches continuum) |

**Outstanding question:** What is the role of CPT in relating quantum lattice systems and QFT?