LSM anomalies of modulated symmetries

- based on SP, Danny Bulmash arXiv: 2602.XXXXX

Motivation

Anomalies are a fundamental property of a quantum theory. (Part of a quantum theory's DNA)

- Many different types : 't Hooft, ABJ, blend, LSM, ...
- Exist in both quantum lattice systems and QFT.

Anomalies of lattice systems $\xrightarrow{\text{map}}$ Anomalies of relativistic QFTs

↑ not anomaly matching

Many, many interesting questions!

Which anomalies are intrinsic to lattice systems? (Kernel of map)
Large class are:

LSM anomalies := obstructions to SPTs with crystalline symmetries

- Best understood for non-gen Sym with group

$$G = G_{int} \times G_{cr}$$

internal Sym group ↲ ↳ crystalline sym group

- More generally, group extention

$$1 \longrightarrow G_{int} \longrightarrow G \longrightarrow G_{cr} \longrightarrow 1$$

When extension splits, $G = G_{int} \rtimes G_{cr}$

- G_{cr} action on G_{int} characterized by homomorphism

$$\rho : G_{cr} \longrightarrow \mathrm{Aut}(G_{int})$$

$$g_{cr} \longmapsto \rho_{g_{cr}}$$

- when ρ is nontrivial, called a G_{int} Modulated Sym

Modulated syms are interesting:

- SSB: goldstones in 1+1 D
- Kramers-Wannier sym: Non-inv spatial reflections
- Gauge theory: Fractons and UV/IR mixing

⋮

also: slow thermalization, tilted optical lattices of cold atoms

This talk: LSM anomalies of modulated sym

- Restrict to bosonic systems in 1+1D with finite G_{int} and G_{cr} sym is lattice transl.

Set up

Spatial lattice of L sites on a ring

- $\mathcal{H} = \bigotimes_{j=1}^{L} \mathcal{H}_j$ with finite dim $\mathcal{H}_j \cong \mathcal{H}_1$

Consider $G_{int} \rtimes \mathbb{Z}_L$ sym ops $\left(\rho : \mathbb{Z}_L \to \mathrm{Aut}(G_{int})\right)$

$$U_g = \prod_{j=1}^{L} U_g^{(j)} \qquad (g \in G_{int})$$

acts on $\mathcal{H}_j$.

$$T^n \qquad (n \in \mathbb{Z}_L)$$

with
$$U_g^{(j)} U_h^{(j)} U_{gh}^{(j)\dagger} = \nu_j(g,h) \in U(1) \qquad (1)$$
$$T U_g^{(j)} T^\dagger = U_{\rho_1(g)}^{(j-1)} \qquad (2)$$

- From (2), satisfy $T U_g T^\dagger = U_{\rho_1(g)}$

LSM anomalies arise from $[\nu_j] \in H^2(G_{int}, U(1))$

- conjugate (1) by T : $\nu_j(g,h) = \nu_{j-1}(\rho_1(g), \rho_1(h))$

$$\Rightarrow \qquad [\nu_j] = \rho_j^*[\nu]. \qquad \text{(translation covariance)}$$

When does $[\nu]$ lead to an LSM anomaly?

Adding anomaly-free ancillas

$$\mathcal{H}_j \longmapsto \mathcal{H}_j \otimes V \otimes W$$

$$U_g \longmapsto \prod_{j=1}^{L} U_g^{(j)} \otimes V_g^{(j)} \otimes W_g^{(j)}$$

$$T \longmapsto T \otimes T_v \otimes T_w$$

- LSM anomaly-free : proj reps $[\gamma_j]$ of $V_g^{(j)}$ and $[\mu_j]$

of $W_g^{(j)}$ Satisfy

$$[\gamma_j] + [\mu_j] = 0$$

- $G_{int} \rtimes \mathbb{Z}_L$ sym: $T_{v/w}$ acts on ancillas as (2)

$$\Rightarrow \quad [\gamma_j] = \rho_j^*[\gamma] \qquad [\mu_j] = \rho_j^*[\mu]$$

translation preserving

Now deform each $V_g^{(j)} \longmapsto V_g^{(j+1)}$

- Causes $[\nu_j] \longmapsto [\nu_j] + [\gamma_{j+1}] + [\mu_j]$

$$\Rightarrow \quad [\nu] \longmapsto [\nu] + (\rho_1^* - 1)[\gamma]$$

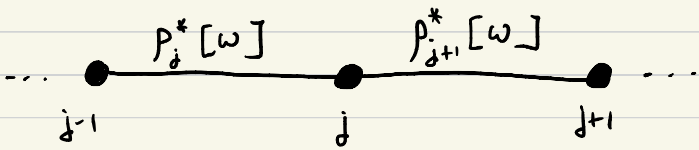
If $[\nu] \in \mathrm{Im}(\rho_1^* - 1) \Rightarrow G_{int} \rtimes \mathbb{Z}_L$ is LSM anomaly-free

↳ Sym ops can be made manifestly anomaly-free using ancillas.

Claim: LSM anomaly free iff $[\nu] \in \mathrm{Im}(\rho_1^* - 1)$

consider real-space construction for a (1+1)D $G_{int} \rtimes \mathbb{Z}_L$ SPT:

→ Each link $\langle j-1, j \rangle$ decorated by (1+1)D SPT $\rho_j^*[\omega] \in H^2(G_{int}, U(1))$



(Remark: Ignoring weak SPT contributions)

→ To produce an SPT, cannot have anomalies @ sites j:

$$[\nu_j] + \rho_j^*[\omega] = \rho_{j+1}^*[\omega]$$

$\Rightarrow$ SPT iff $[\nu] = (\rho_1^* - 1)[\omega]$

LSM anomalies classified by

$[\nu] \sim [\nu] + (\rho_1^* - 1)[\gamma] \Rightarrow \quad H^2(G_{int}, U(1)) / Im(\rho_1^* - 1)$

→ For infinite lattice, $G = G_{int} \rtimes \mathbb{Z}$ and $\rho : \mathbb{Z} \to Aut(G_{int})$:

$$\frac{H^2(G_{int}, U(1))}{Im(\rho_1^* - 1)} = H^1(\mathbb{Z}, \underset{\substack{\uparrow \\ \text{nontrivial } \mathbb{Z}\text{-module}}}{H^2(G_{int}, U(1))}) \subset H^3(G_{int} \rtimes \mathbb{Z}, U(1))$$

$\Rightarrow$ agrees with crystalline equivalence princp. expectations

→ Consequences:

ρ	$[\nu]$	LSM anomaly
trivial	$= 0$	No
trivial	$\neq 0$	Yes
nontrivial	$= 0$	No
nontrivial	$\neq 0$	Maybe

Example 1: $\mathbb{Z}_N$ dipole sym

$\mathbb{Z}_N \times \mathbb{Z}_N$ modulated sym with $\rho_n((g_1, g_2)) = (g_1 + n g_2, g_2)$

→ Finite lattice requires $L = 0 \mod N$

→ can show $\rho_1^* [\nu] = [\nu] \quad \forall \; [\nu] \in H^2(\mathbb{Z}_N^2, U(1))$

$\Rightarrow$ any $[\nu] \neq 0$ leads to an LSM anomaly.

Example 2: Exponential Sym

$\mathbb{Z}_N \times \mathbb{Z}_N$ modulated sym with $\rho_n((g_1, g_2)) = (a^n g_1, b^n g_2)$

→ Integers a, b coprime to N

→ Finite lattice requires $a^L = b^L = 1 \mod N$

→ can show $\rho_1^* [\nu] = ab[\nu]$

let $[\nu] = K[\alpha]$ with $[\alpha]$ a generator of $H^2(\mathbb{Z}_N^2, U(1))$
with 2-cocycle $\alpha(g,h) = \exp\left[\frac{2\pi}{N} i g_1 h_2\right]$

→ LSM anom-free cond $[\nu] \in \mathrm{Im}(\rho_1^* - 1)$ satisfied
iff $\gcd(ab-1, N) \mid K$.

Case	LSM anomaly for $[\nu] \neq 0$ ($K \neq 0 \mod N$)?
$ab = 1 \mod N$	Always
$\gcd(ab-1, N) = 1$	Never
$\gcd(K, N) = 1$	iff $\gcd(ab-1, N) \neq 1$

Outlook

LSM anomalies of modulated sym w lattice transl in 1+1D

→ LSM anomaly existence depends on onsite proj rep and homomorphism ρ.

What else is in arXiv: 2602.XXXXX

1) Examples of 1+1D stabilizer code models

   - SPT-LSM theorems

2) LSM anomalies of $G_{int} \rtimes pm$

3) Beyond 1+1D

   - Stratified anomalies: generalization of onsite proj reps

4) Examples in 2+1D.