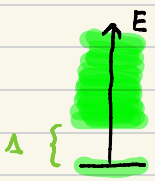
Symmetry-enforced Fermi Surfaces

Based on 2512:04150 (PRL '26) with Luke Kim and Shu-Heng Shao

Landscape of quantum phases

Phases of quantum matter come in two flavors
(distinguished by many-body energy spectrum)

1) gapped phases: finite bulk energy gap seperating ground states from excited states



→ e.g., Insulators, paramagnets, Quantum Hall phases

→ Systematically understood: discrete SSB, SPT, Topological order

2) gapless phases: No energy gap ($\Delta = 0$)

→ e.g., Superfluids, Ferromagnets, Metals

→ Much less understood than gapped phases.
(Desirable to have a more Systematic understanding)

→ Fruitful approach: Find microscopic constraints that forbid gapped phases.

Symmetry-enforced gaplessness

Forbid gapped phases by enforcing a microscopic, exact symmetry.

Closely related to Lieb-Schultz-Mattis theorems and 't Hooft anomalies

→ Always forbid Symmetry-preserving, trivially gapped phases

→ Generally compatible with gapped phases with SSB / topological order

→ Can sometimes enforce gaplessness

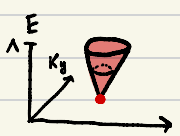
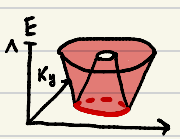
Weak version: Symmetry + an IR constraint ⇒ gaplessness

	Example	dimension	Symmetry	IR constraint
't Hooft anomaly →	Wang-Senthil '14	2	$SU(2) \times \mathbb{Z}_4^T / \mathbb{Z}_2$	No $\mathbb{Z}_4^T$ SSB
No anomaly →	Watanabe '17	≥ 1	$U(1)$	Compressibility

Strong version: Symmetry ⇒ gaplessness

→ eg, perturbative 't Hooft anomalies (Córdova, Freed, Teleman '24)

→ Known examples enforce finitely many gapless modes.

# gapless Modes	Spectrum	Example
finite		Superfluid
∞		Fermi Surface

→ This talk: demonstrate Symmetry ⇒ Fermi surface

Set up

Consider $L_x \times L_y$ square lattice with PBCs and one spinless

fermion per site:

$$\{c_r^\dagger, c_{r'}\} = \delta_{r,r'} \qquad \{c_r, c_{r'}\} = 0$$

→ Assume finite-range Hamiltonian $H = \sum_r H_r$

Goal: Find unitary symmetry operators $\{U_g\}_{g \in G}$ satisfying

$$U_g U_h = U_{gh} \qquad (g, h \in G)$$

$$[U_g, H] = 0 \implies H \text{ has a FS}$$

→ G symmetry is a sufficient, but not necessary condition for a Fermi surface

The Symmetry

Must forbid all terms that can destroy a Fermi surface
(Fermi surface pacifist joke)

→ For example: $-\mu \sum_r n_r$ (chemical potential term)

$\sum_{\langle r,r' \rangle} (\Delta c_r c_{r'} + hc)$ (Pairing terms)

$\sum_{\langle r,r' \rangle} n_r n_{r'}$ (density-density interactions)

We'll enforce the U(1) symmetry $\{e^{i\theta N}\}$ with $N = \sum_r n_r$

$$e^{i\theta N} c_r e^{-i\theta N} = e^{-i\theta} c_r$$

→ rules out pairing term

To forbid other terms, consider the real fermion operators

$$a_r = c_r^\dagger + c_r \qquad b_r = i(c_r^\dagger - c_r)$$

$$\{a_r, a_{r'}\} = \{b_r, b_{r'}\} = 2\delta_{r,r'} \qquad \{a_r, b_{r'}\} = 0$$

$$\longrightarrow n_r = (i a_r b_r + 1)/2$$

$\Rightarrow$ can forbid number operator terms by forcing a_r and b_r to decouple

Majorana translation symmetry

$$T_v^{(b)} \begin{pmatrix} a_r \\ b_r \end{pmatrix} T_v^{(b)\dagger} = \begin{pmatrix} a_r \\ b_{r+v} \end{pmatrix}$$

$\longrightarrow$ By locality, if $T_v^{(b)} H = H T_v^{(b)}$,

$$H = H_a(a_r) + H_b(b_r) \quad \} \text{ Theory factorizes.}$$

$\longrightarrow$ complex fermions transform as

$$T_v^{(b)} c_r T_v^{(b)\dagger} = \tfrac{1}{2}(c_r^\dagger + c_r - c_{r+v}^\dagger + c_{r+v})$$

(mixes particle and hole states)

Symmetric Hamiltonians

Every finite-range H satisfying $[H, N] = [H, T_v^{(b)}] = 0$ takes the form

$$H = \frac{i}{4} \sum_{r,v} (a_r a_{r+v} + b_r b_{r+v}) \qquad (g_{-v} = -g_v \in \mathbb{R})$$

$$= i \sum_{r,v} g_v c_r^\dagger c_{r+v}$$

→ why?

1) Majorana translations decouple a_r and b_r.

2) U(1) symmetry transformation mixes a_r and b_r

$$\begin{pmatrix} a_r \\ b_r \end{pmatrix} \longmapsto \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_r \\ b_r \end{pmatrix}$$

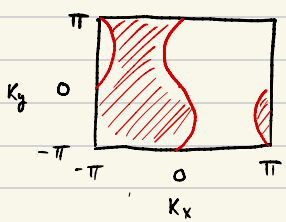
⇒ Symmetric terms with a_r and b_r decoupled are quadratic

→ Symmetry + locality ⇒ Non-interacting fermions

→ Single particle dispersion $\omega_K = -\sum_v g_v \sin(K \cdot v)$

For example $H = -\frac{i}{2} \sum_r \left(c_r^\dagger c_{r+\hat{x}} + \frac{3}{4} c_r^\dagger c_{r+\hat{y}} \right) + h.c.$

→ $\omega_K = \sin(K_x) + \frac{3}{4}\sin(K_y)$ and has Fermi surface



It follows from $\omega_{-K} = -\omega_K$ that

1) Every ground state is at half-Filling

2) Every H has a Fermi surface.

→ Regions with $\omega_K > 0$ and $\omega_K < 0$ related by inversion

→ Boundary separating these regions is a curve with $\omega_K = 0$ ⇒ a Fermi surface.

3) A generic Fermi Surface is topologically non-trivial

→ Each point $K \equiv -K$ lies on a non-contractible component of the Fermi Surface

Microscopic Symmetry operators do not commute: $[N, T_v^{(b)}] \neq 0$

→ Let $Q = N - L_x L_y/2$:

$$T_v^{(b)} Q T_v^{(b)\dagger} = \frac{i}{2} \sum_r a_r b_{r+v} \equiv Q_v$$

→ Full microscopic Symmetry group is an infinite-dimensional Lie group related to the Onsager algebra ⇒ Onsager Symmetry.

$$[Q_v, Q_{v'}] = i G_{v'-v} \quad \text{where} \quad G_v = \frac{i}{2} \sum_r (a_r a_{r+v} - b_r b_{r+v})$$

$$[G_v, G_{v'}] = 0$$

$$[Q_v, G_{v'}] = 2i (Q_{v-v'} - Q_{v+v'})$$

→ Not part of the free-fermion Symmetry $c_K \mapsto e^{-i\theta(K)} c_K$.

→ Includes Symmetry transformation

$$c_K \mapsto e^{-if(K)} c_K \qquad (f(K) = f(-K))$$

generated by $e^{i \sum_K f(K) Q_K}$ with $Q_K = \frac{1}{L_x L_y} \sum_v \cos(K \cdot v) Q_v$

→ For K on the Fermi Surface, this forms a subgroup of the LU(1) Symmetry of Else, Thorngren, and Senthil '20.

Summary

U(1) Fermion number + Majorana translation Symmetry

= an Onsager Symmetry $\Rightarrow$ Fermi Surface

→ Focused on Square lattice, but conclusion holds for any d-dimensional Bravais lattice

Two open questions:

1) Stability to ancillae

2) Generalizations to interacting theories.