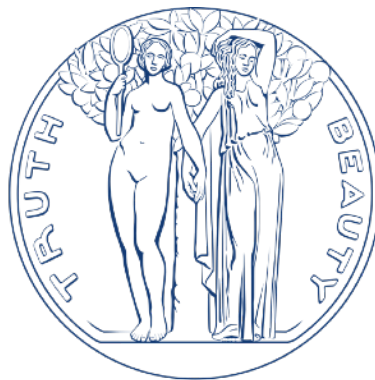


When Symmetry Does the Unexpected

Sal Pace

IAS Princeton



Bard College Seminar

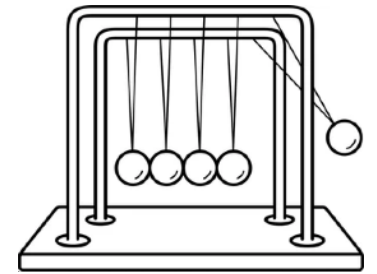
The Power of Symmetry

Symmetry is a foundation of modern physics

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Classical mechanics: Conservation laws: Energy,
momentum, angular momentum

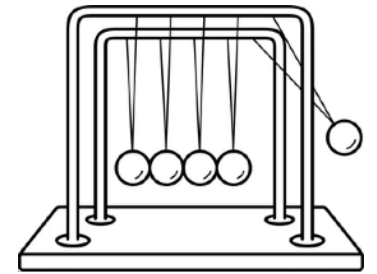
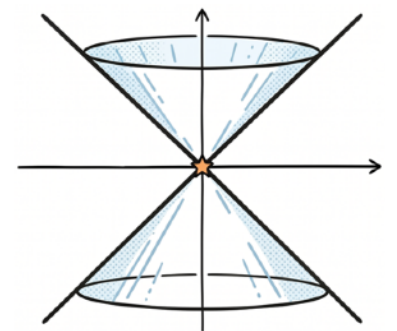


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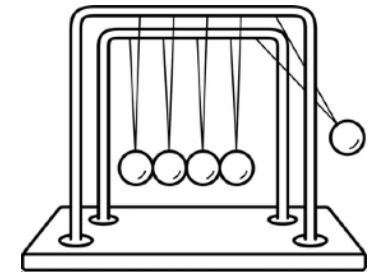
Relativity: Lorentz invariance of spacetime



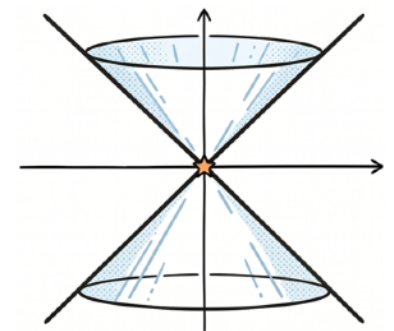
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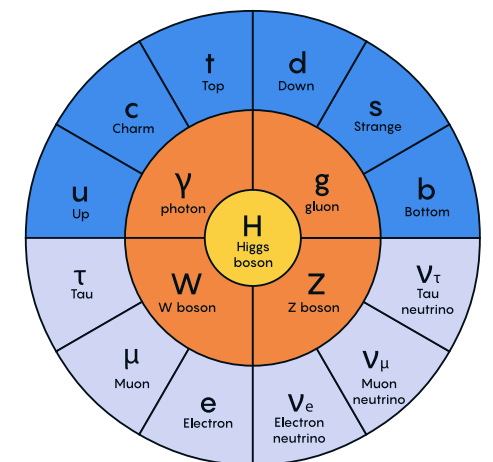
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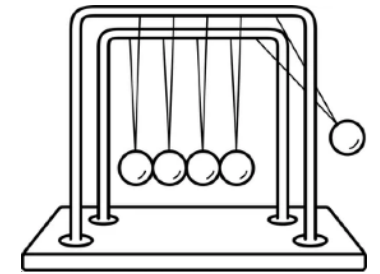
Particle physics: Underlying principle of the Standard Model



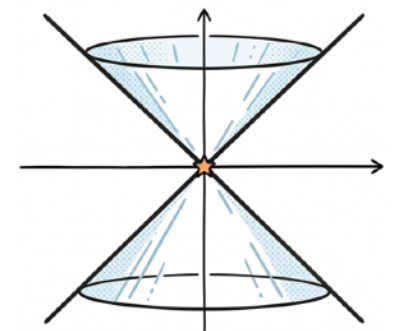
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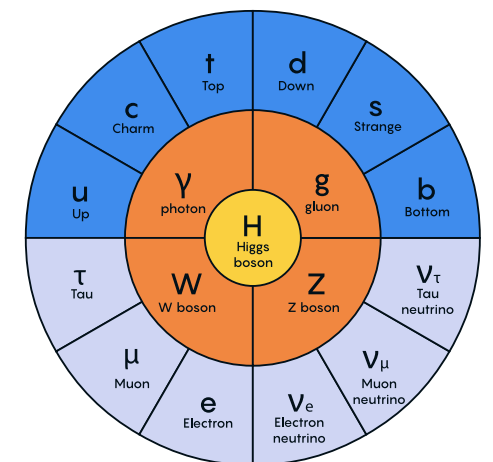
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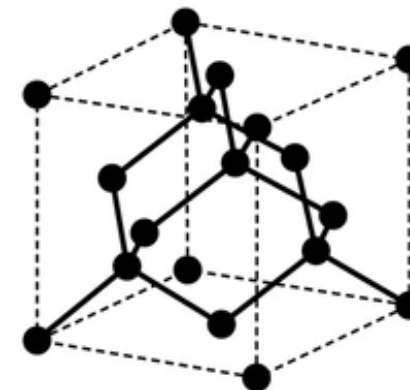
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Particle physics: Underlying principle of the Standard Model



Condensed matter: Phases of matter: Magnets, crystals, superfluids



Two Sides of Symmetry

Symmetry as a tool

Symmetry as a principle

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Symmetry as a **tool**

*Helps us **understand** a system*

- **Simplifies** calculations
- **Explains** degeneracies and selection rules
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Physics



Symmetry in Classical Physics

Symmetry in classical physics transforms classical states

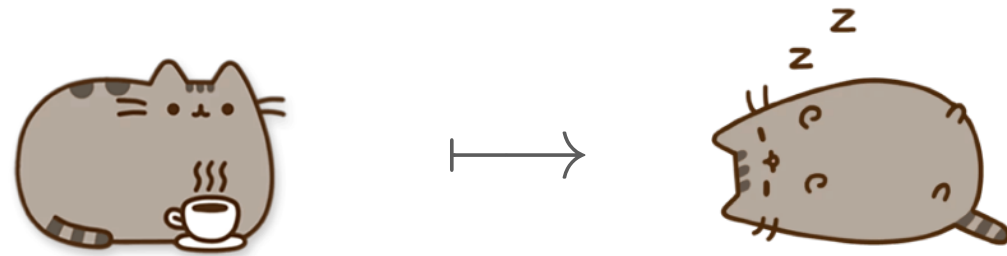
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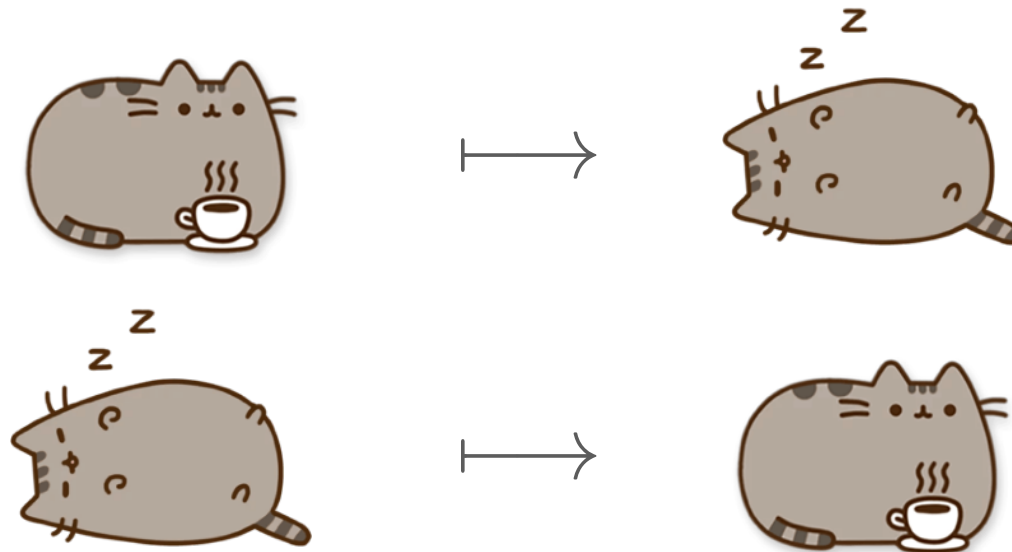
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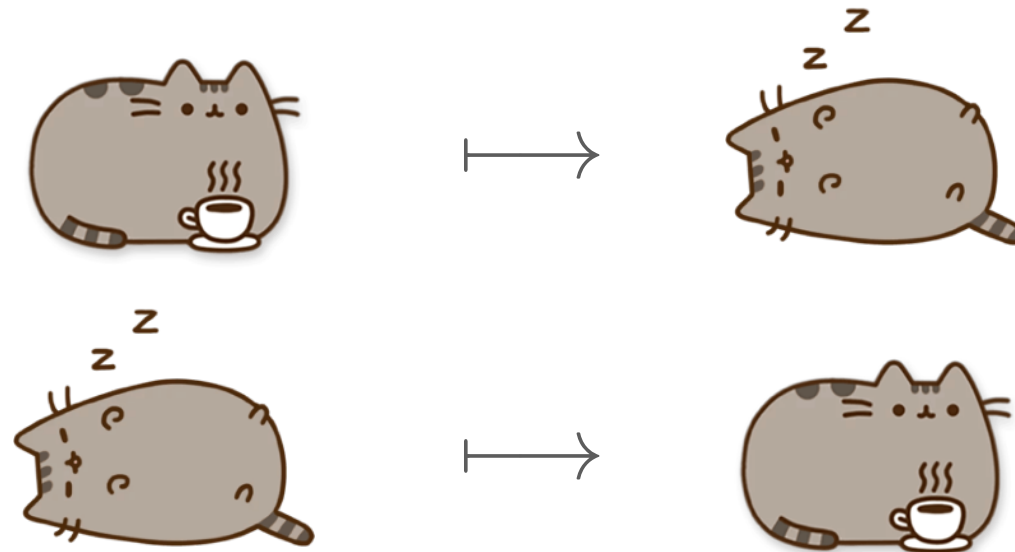
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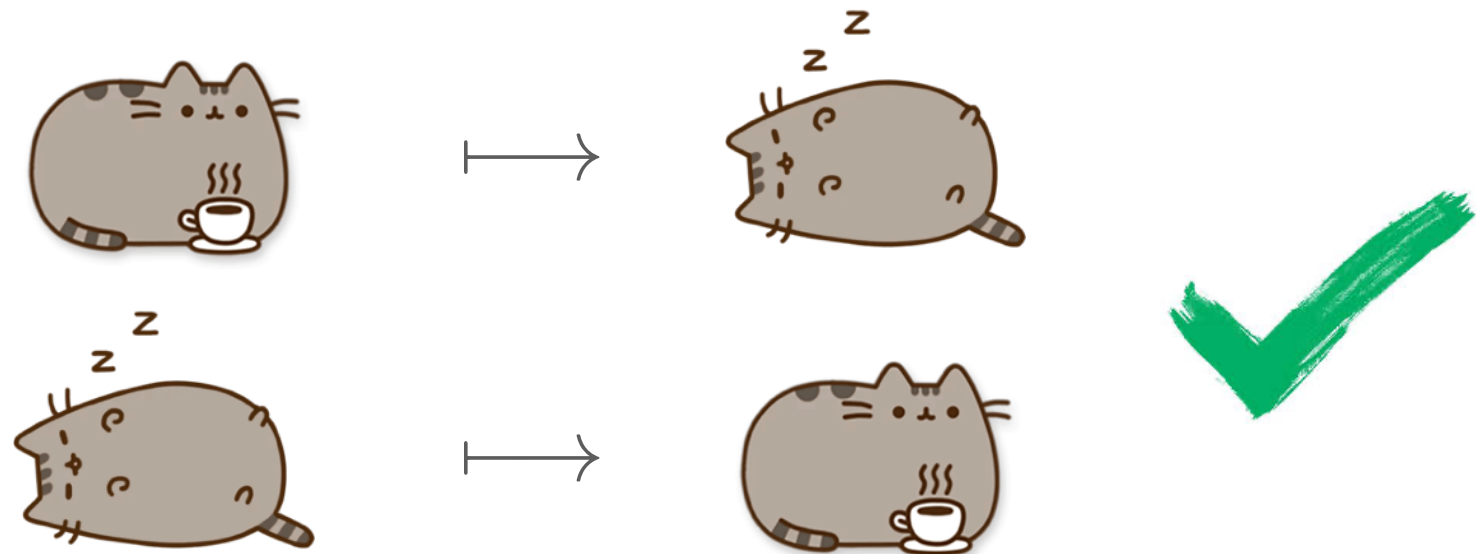
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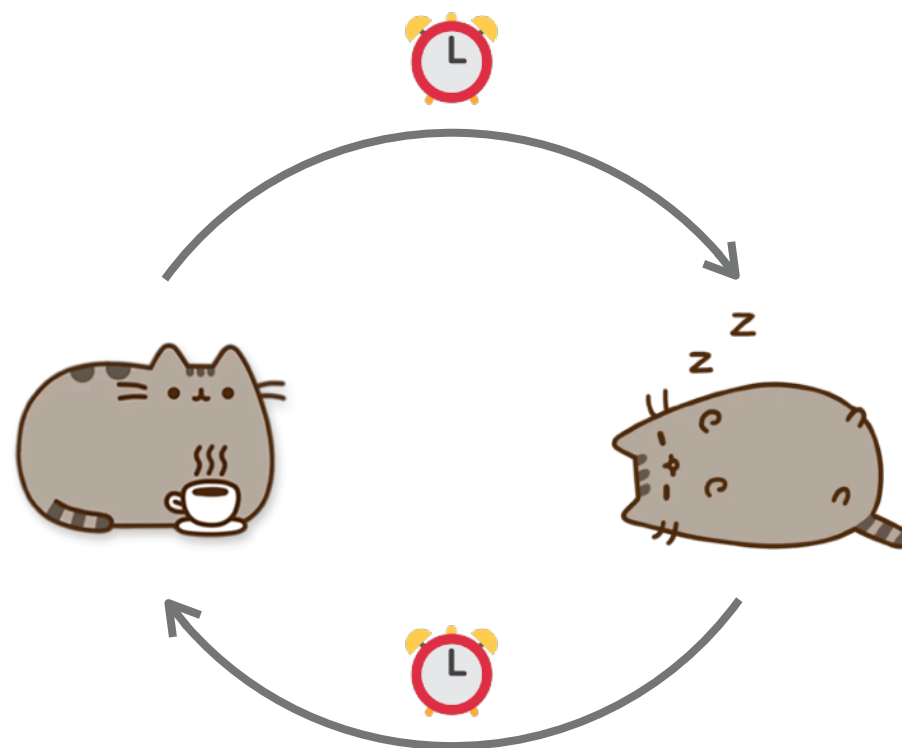
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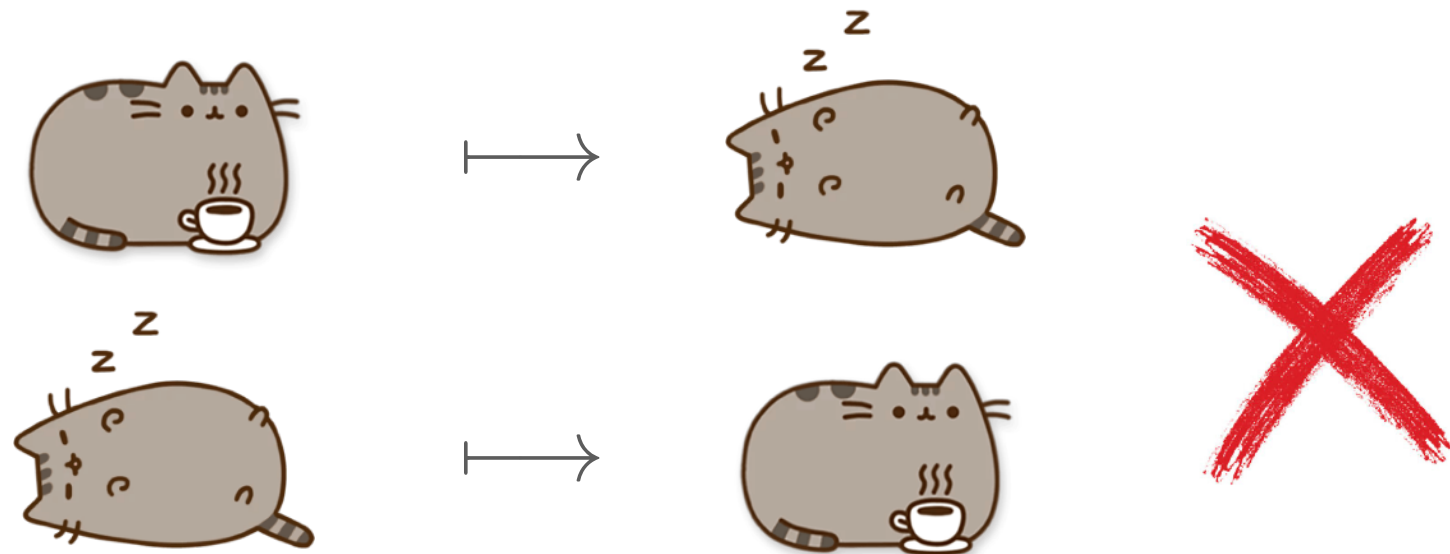
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Example:



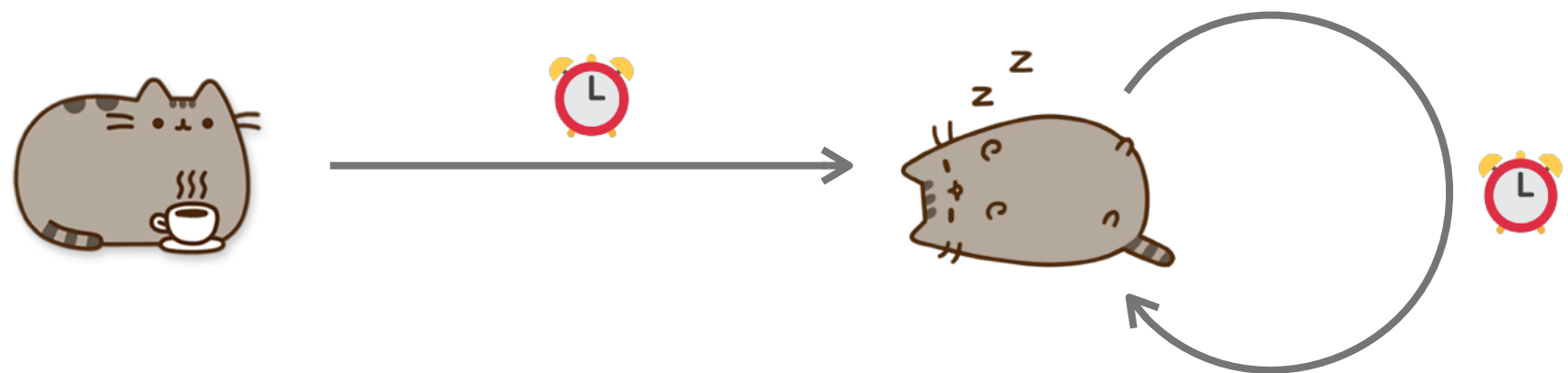
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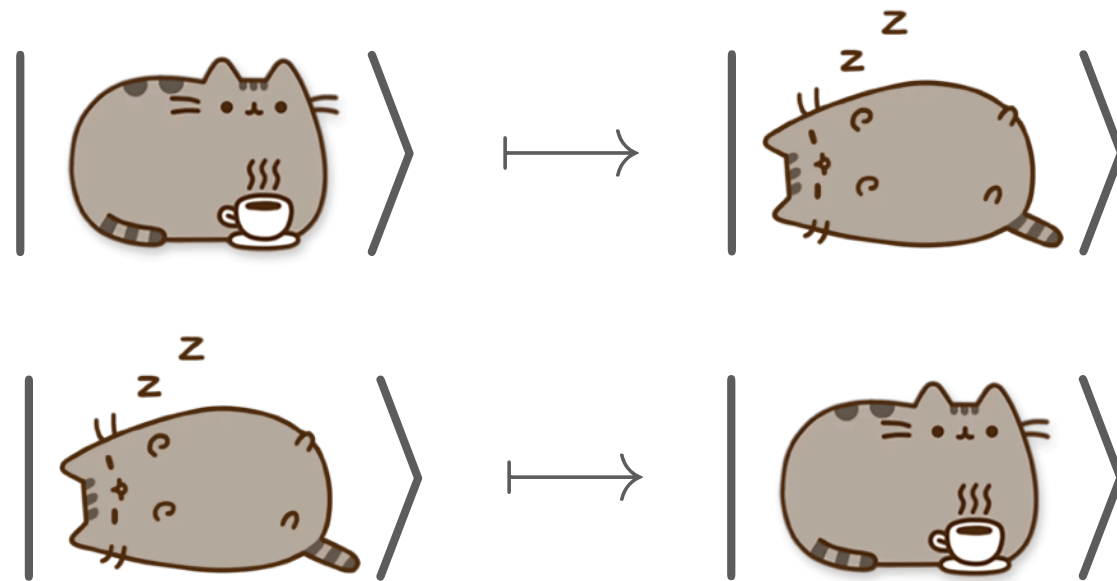


Symmetry in Quantum Physics

Symmetry in quantum physics transforms quantum states

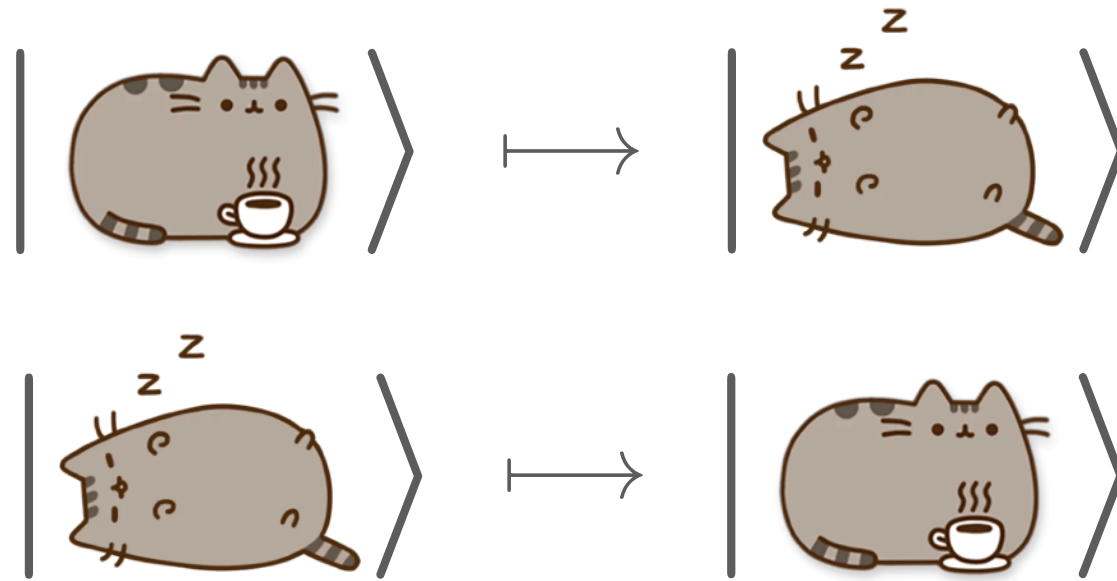
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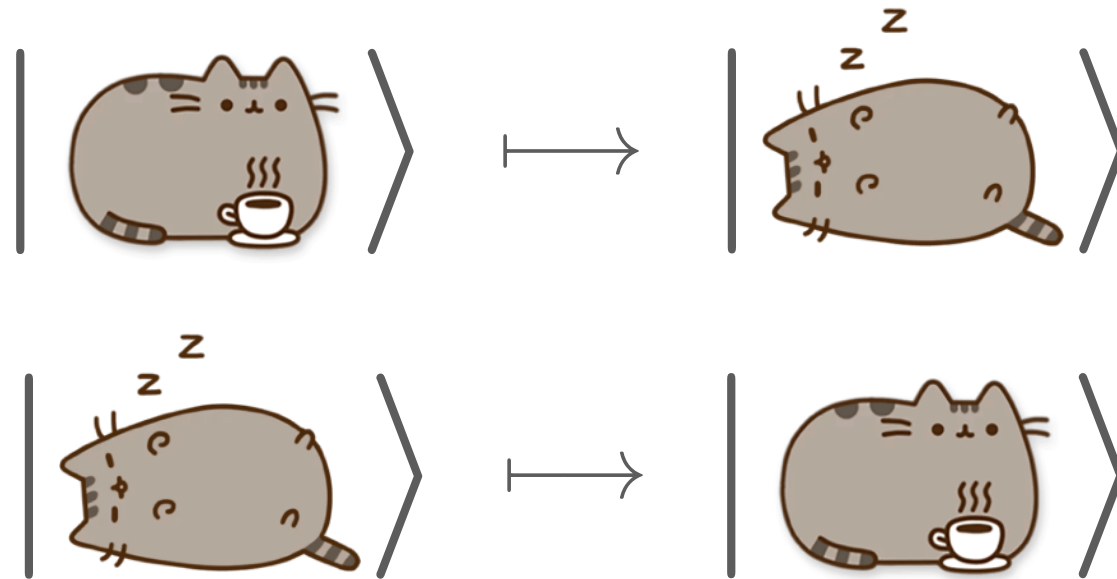
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Two new quantum properties:

1. Superposition
2. Relative phases

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Symmetry in quantum physics transforms quantum states

Example:

$$| \text{cat} \rangle \mapsto \frac{1}{\sqrt{2}} \left(| \text{cat} \rangle + | \text{cat}^z \rangle \right)$$

$$| \text{cat}^z \rangle \mapsto \frac{1}{\sqrt{2}} \left(| \text{cat} \rangle - | \text{cat}^z \rangle \right)$$

(symmetry operator commutes with the Hamiltonian)

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Symmetry is more interesting
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Symmetry of a Spin-1/2

Consider a **quantum** spin-1/2: $|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$

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Consider a quantum spin-1/2: $|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$

► Why? Answer 1

General Hamiltonian:

$$H = \begin{pmatrix} a & b \\ b^* & d \end{pmatrix}$$

$$[H, Z] = 0 \implies b = 0$$

$$[H, X] = 0 \implies a = d$$

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Symmetry of a Spin-1/2

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Why? Answer 2

Within any 1D eigenspace, X and Z must act as c-numbers.

But c-numbers commute $\implies$ There are **no 1D eigenspaces**

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This talk: Overview unexpected consequences of symmetry
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Symmetry principles enforcing nontrivial quantum phenomena

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Symmetry principles enforcing nontrivial quantum phenomena

1) Correlated quantum matter/field theory

Symmetry can constrain low-energy dynamics

2) Quantum entanglement

Symmetry can enforce nontrivial entanglement

3) Generalized symmetry

New symmetry, same consequences

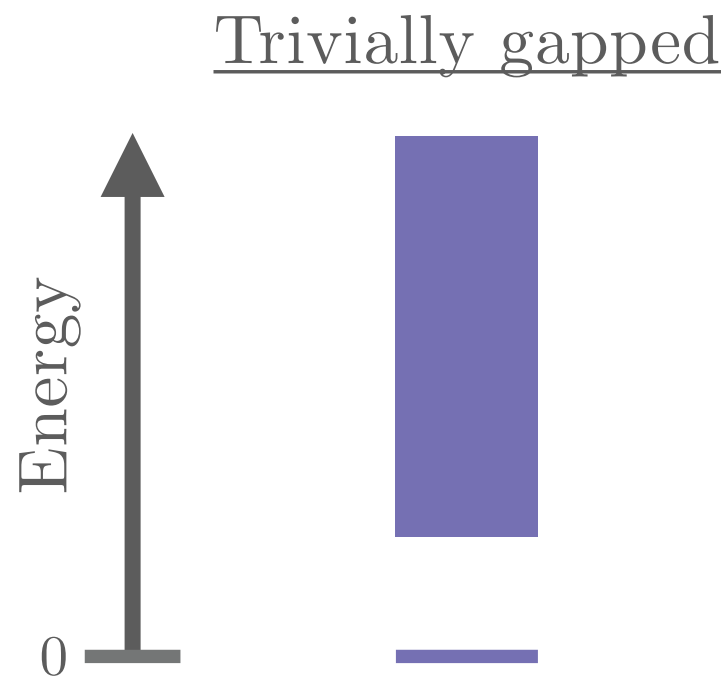
Correlated Quantum Matter/Field Theory

The Grand Challenge

Given a **quantum system** with many interacting degrees of freedom, what is its **low-energy/long-distance** behavior?

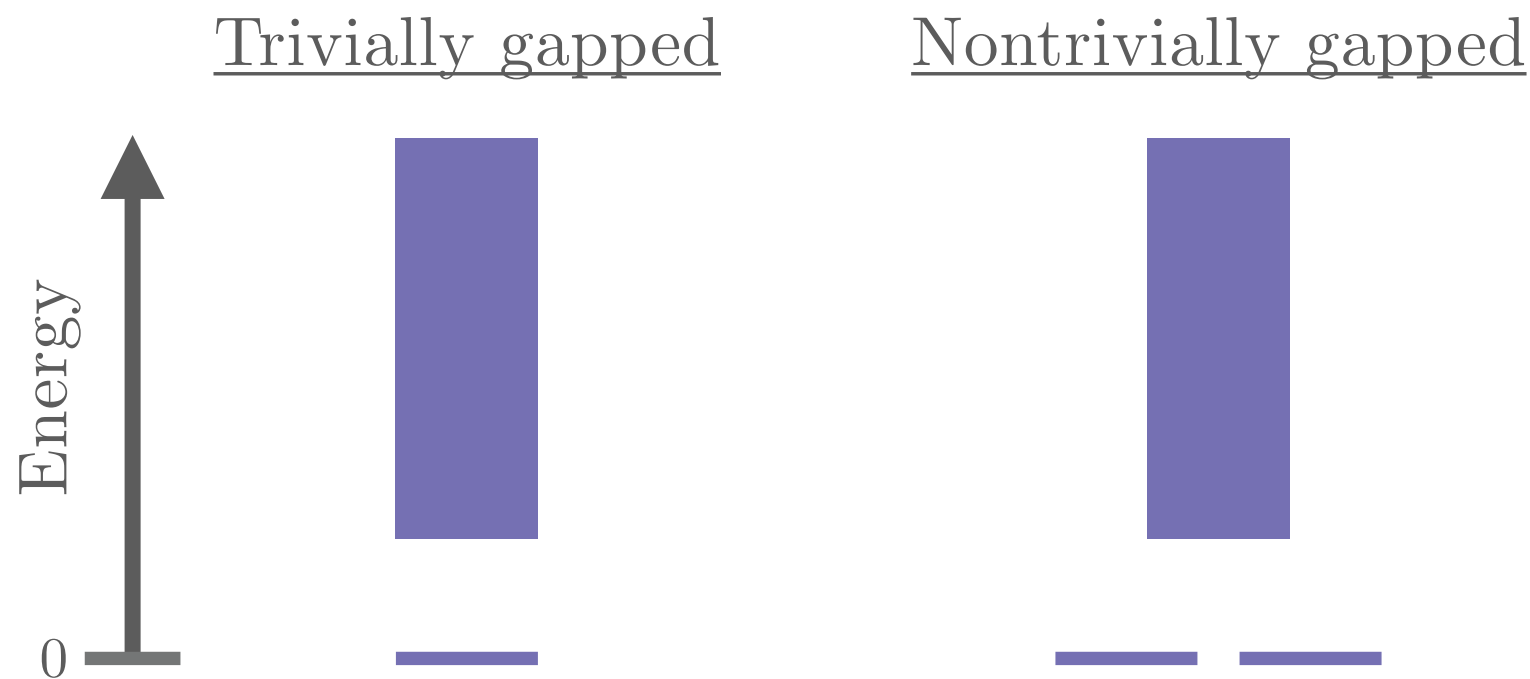
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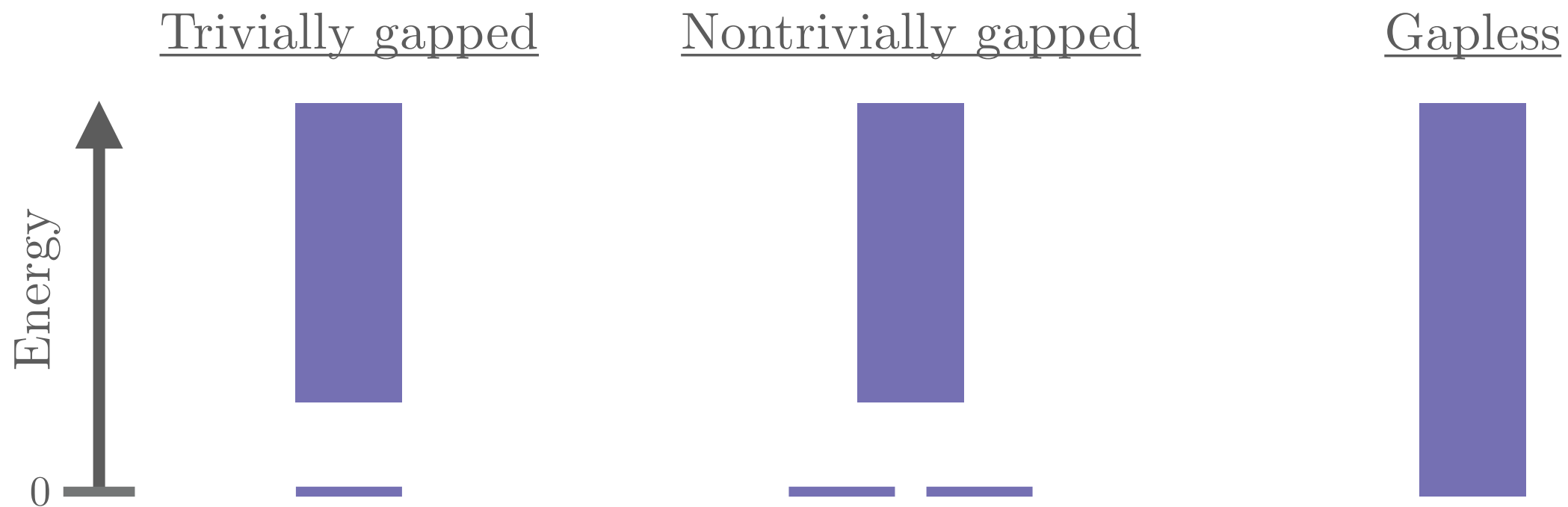
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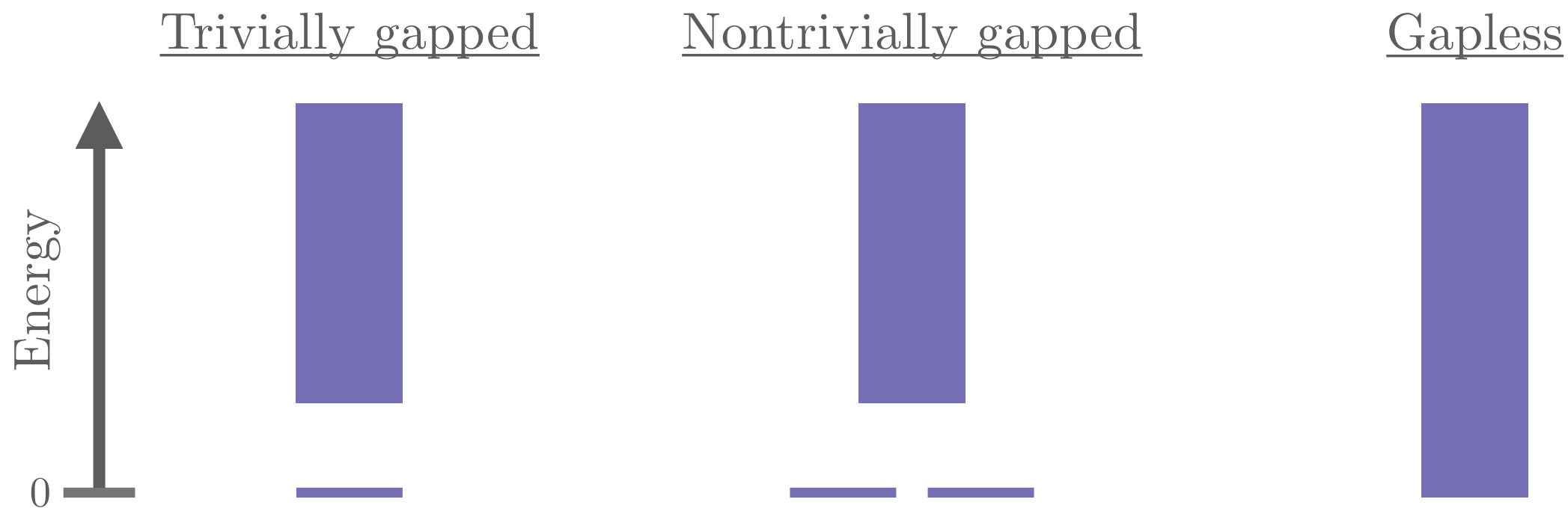
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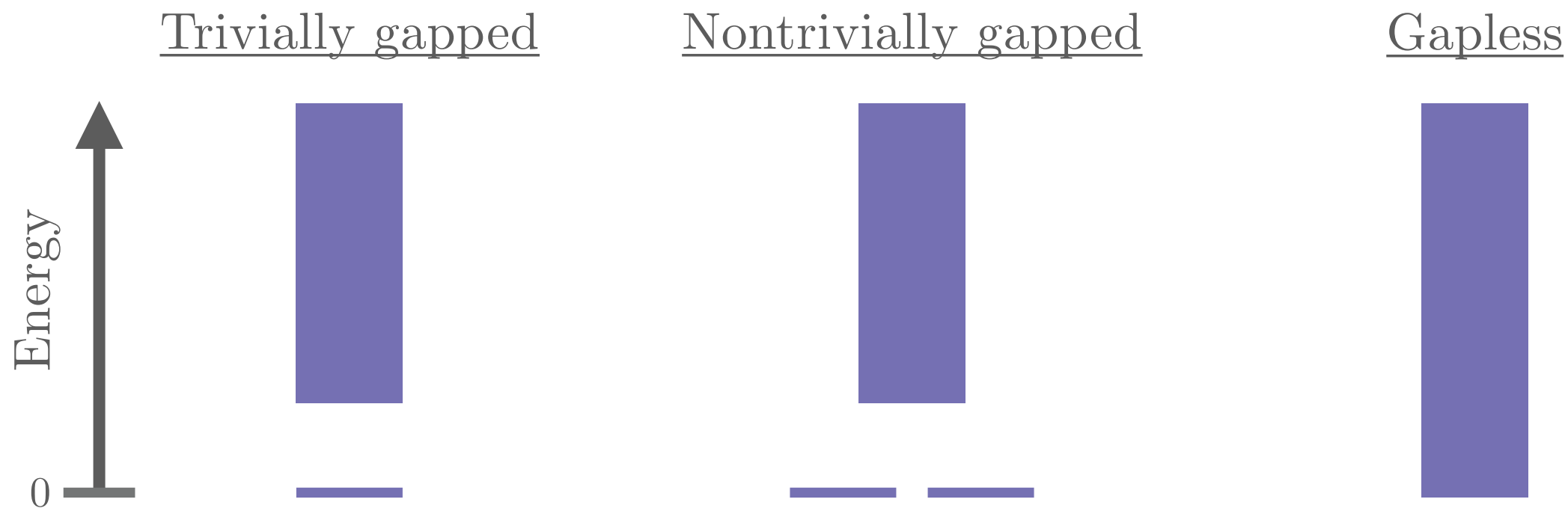
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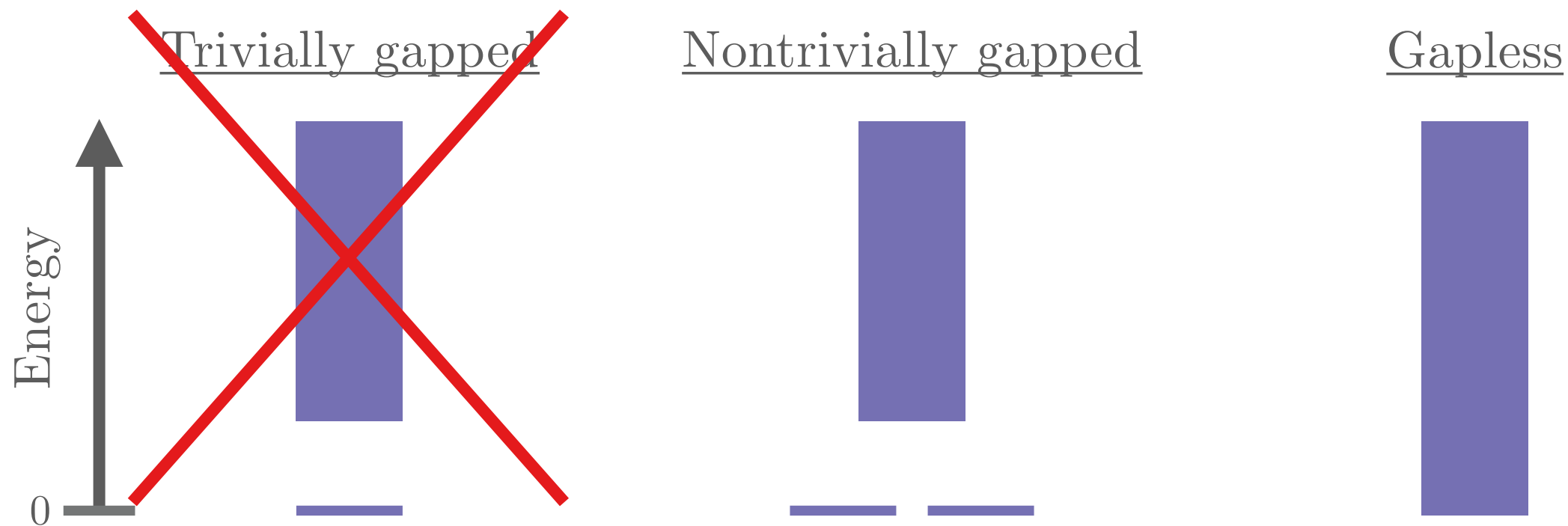


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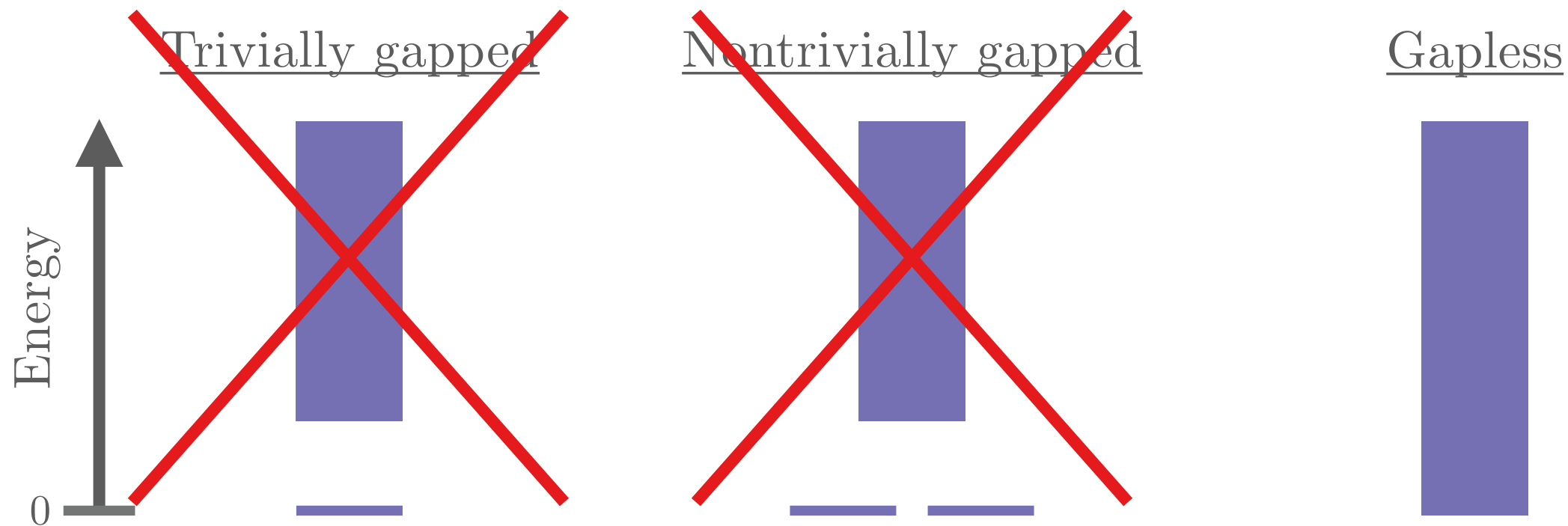


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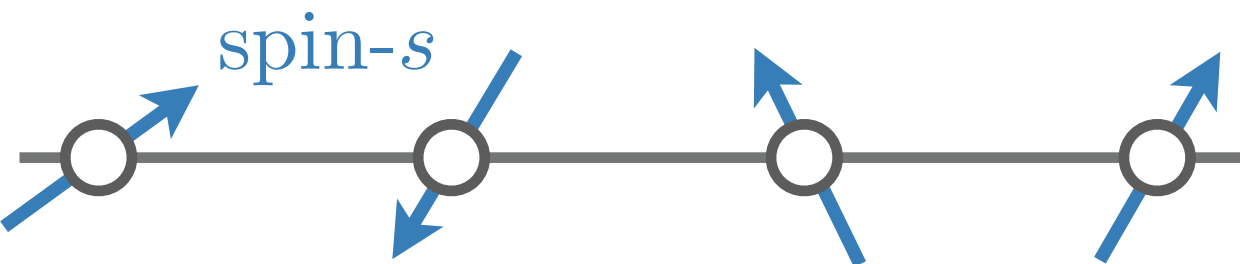


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The Lieb-Schultz-Mattis Theorem

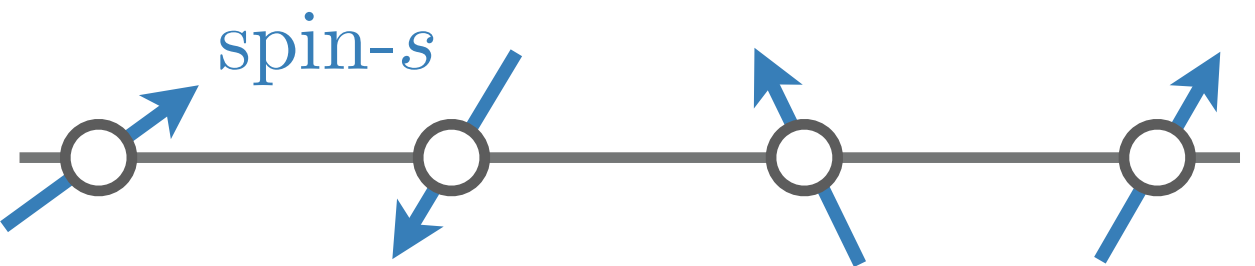
Consider Quantum spin chain



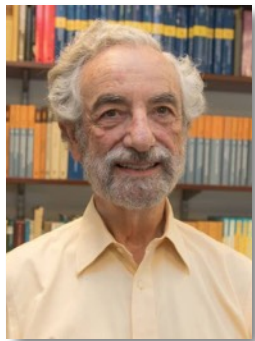
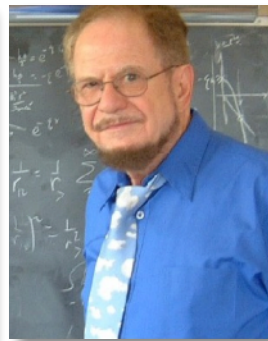
The diagram illustrates a 1D quantum spin chain. It consists of a horizontal line with four white circular nodes. From each node, a blue arrow (vector) extends outwards. The arrows are labeled 'spin-s' in blue text above the second node. The arrows point in different directions: up-right from the first node, down-left from the second, up-left from the third, and up-right from the fourth.

► Symmetry: $\vec{S}_j \mapsto \vec{S}_{j+1}$ and $\vec{S}_j \mapsto R \vec{S}_j$ with $R \in \text{SO}(3)$

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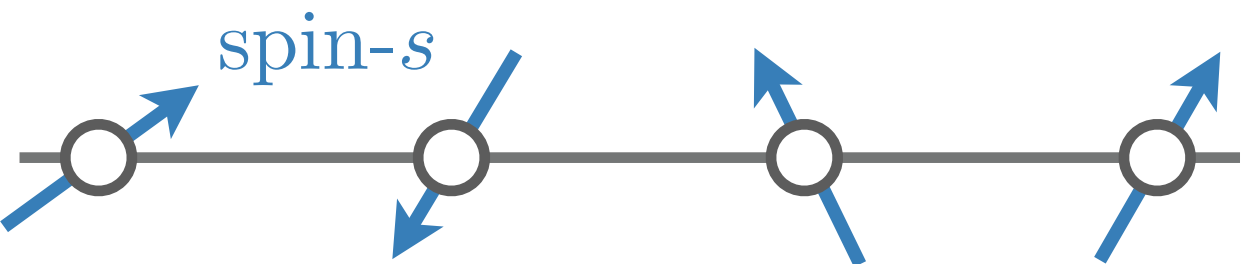
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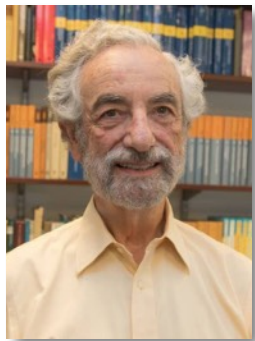
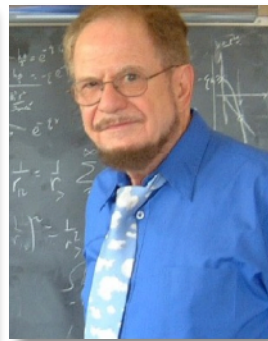


Lieb-Schultz-Mattis 1961: This **symmetry** forbids a **trivial gapped phase** when s is half-integer

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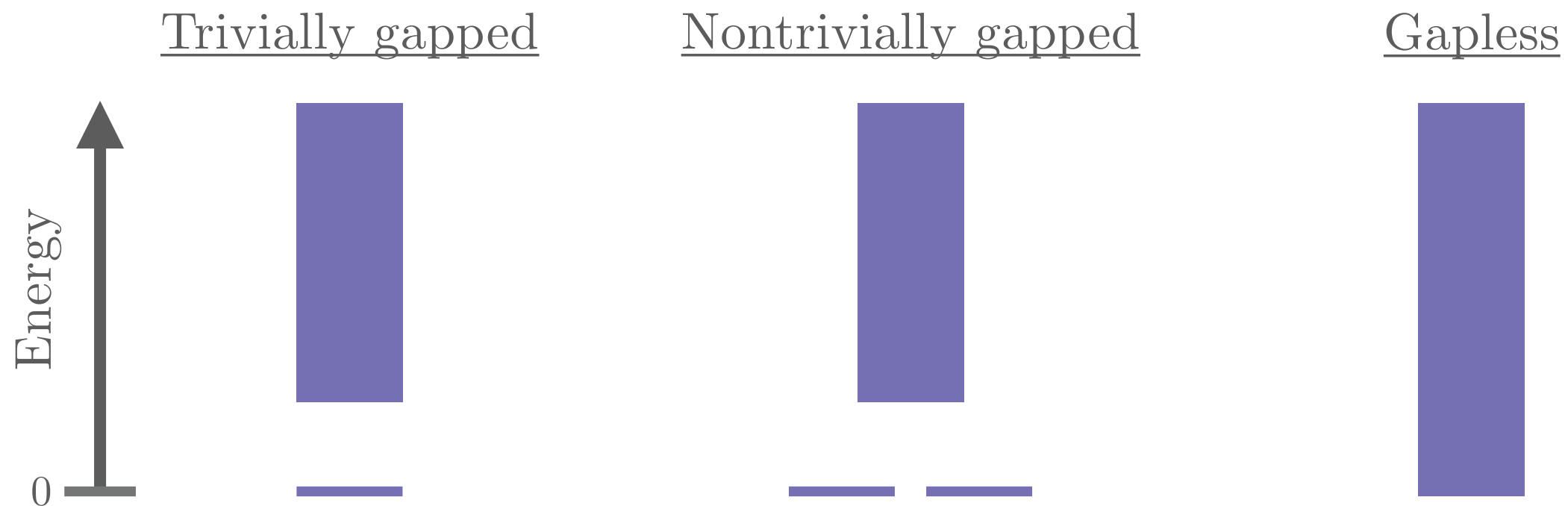
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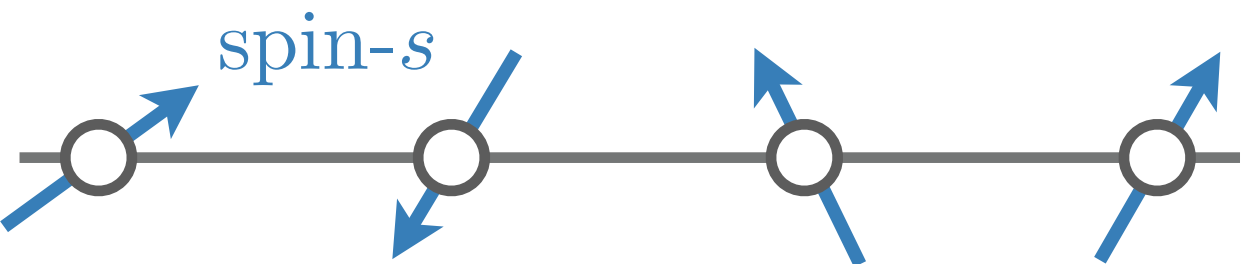


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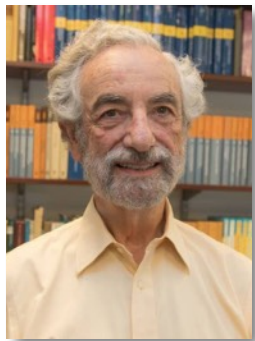
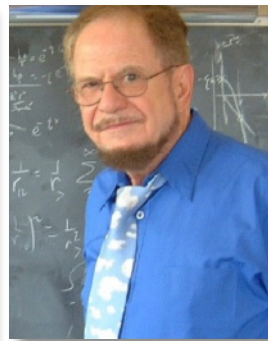
if $s = 1, 2, 3, \dots$



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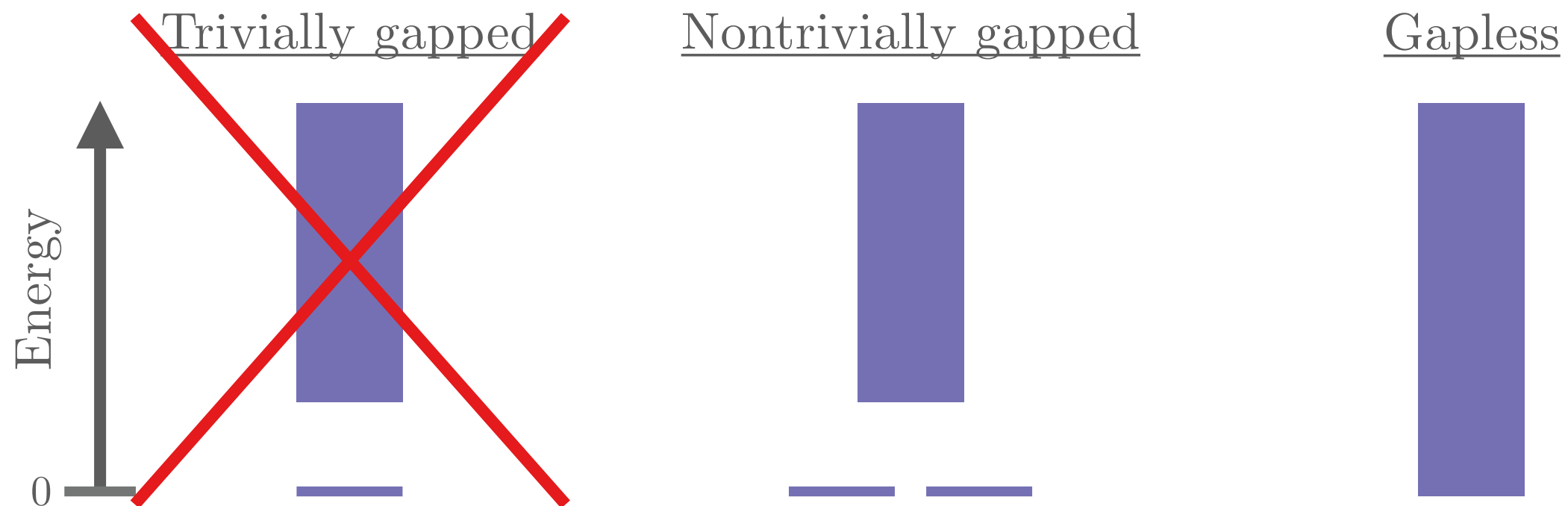
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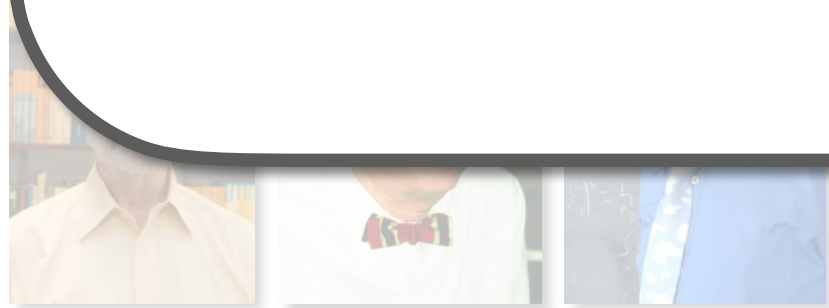


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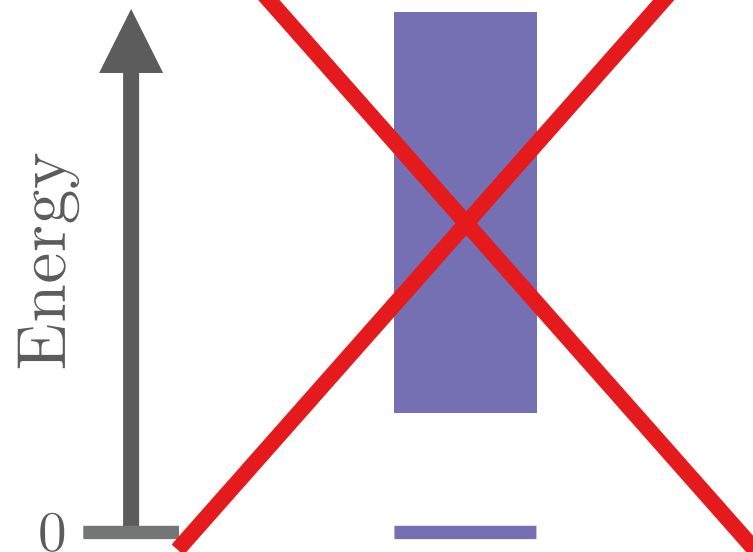
Holds for every symmetric local Hamiltonian!



half-integer

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Trivially gapped



Nontrivially gapped



Gapless



The Lieb-Schultz-Mattis Theorem

Consider Quantum spin chain



Holds for every symmetric local Hamiltonian!

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$

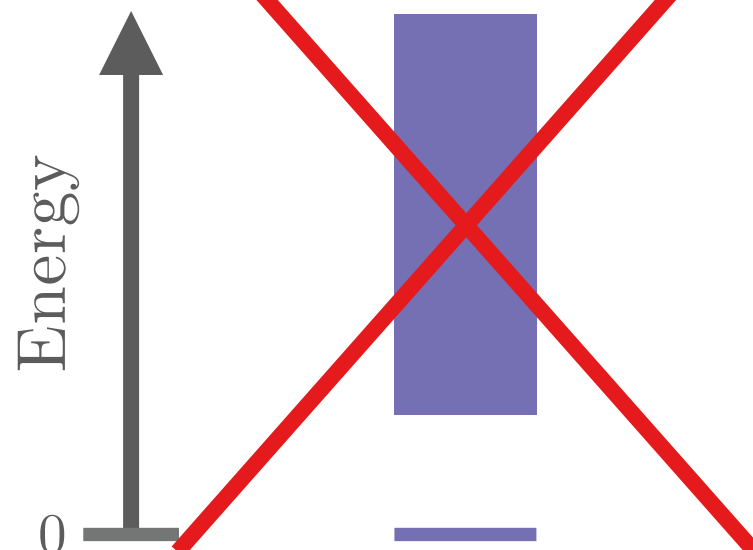
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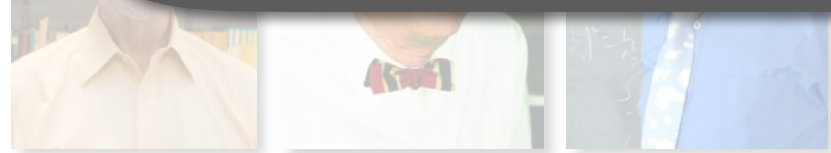


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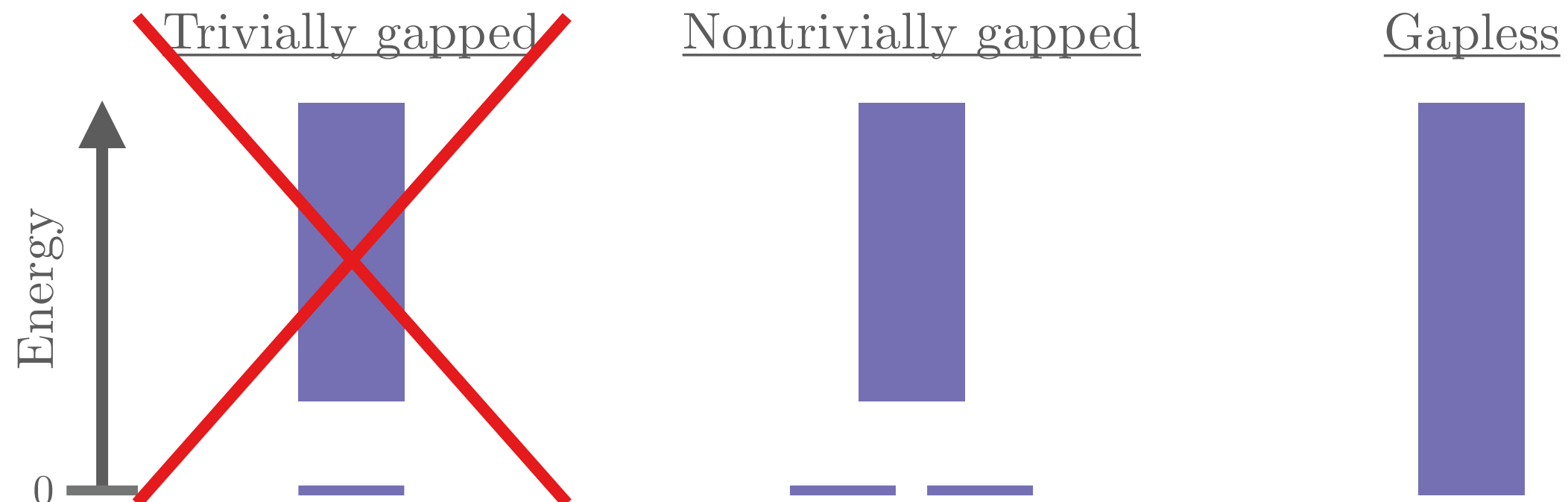
Holds for every symmetric local Hamiltonian!

$$H = \sum_j \left(\vec{S}_j \cdot \vec{S}_{j+1} + 67 \vec{S}_j \cdot \vec{S}_{j+2} - 42 (\vec{S}_j \cdot \vec{S}_{j+1}) (\vec{S}_{j+2} \cdot \vec{S}_{j+3}) \right)$$



half-integer

if $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$

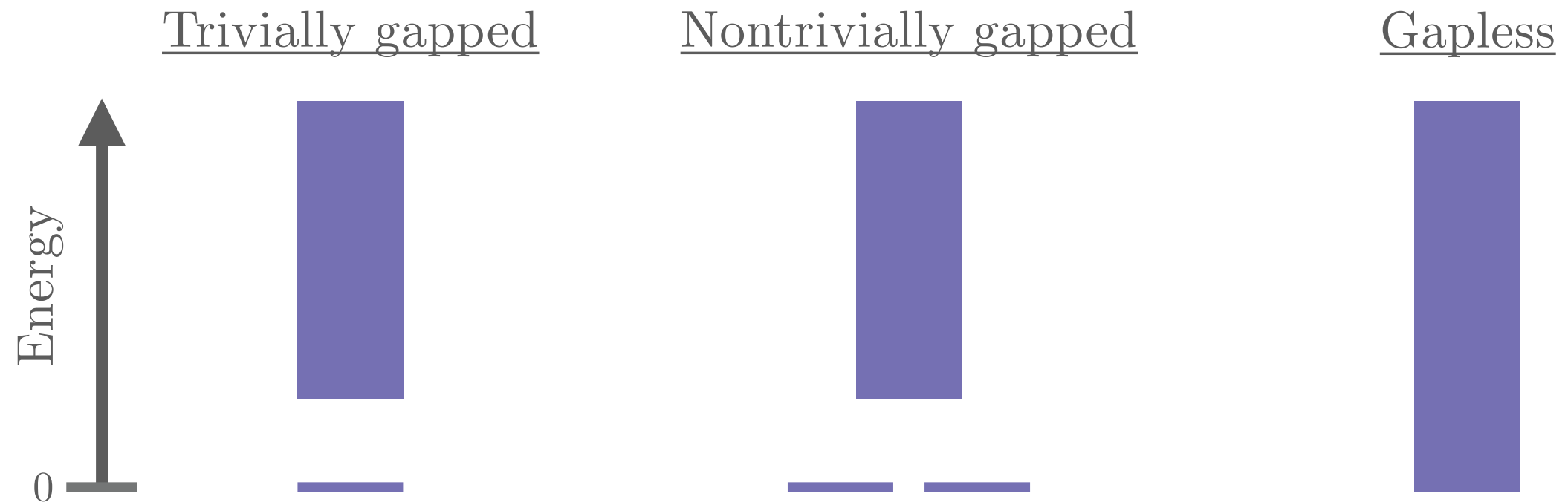


Symmetry-Enforced Gaplessness

Sometimes, *symmetry* is even more *powerful*

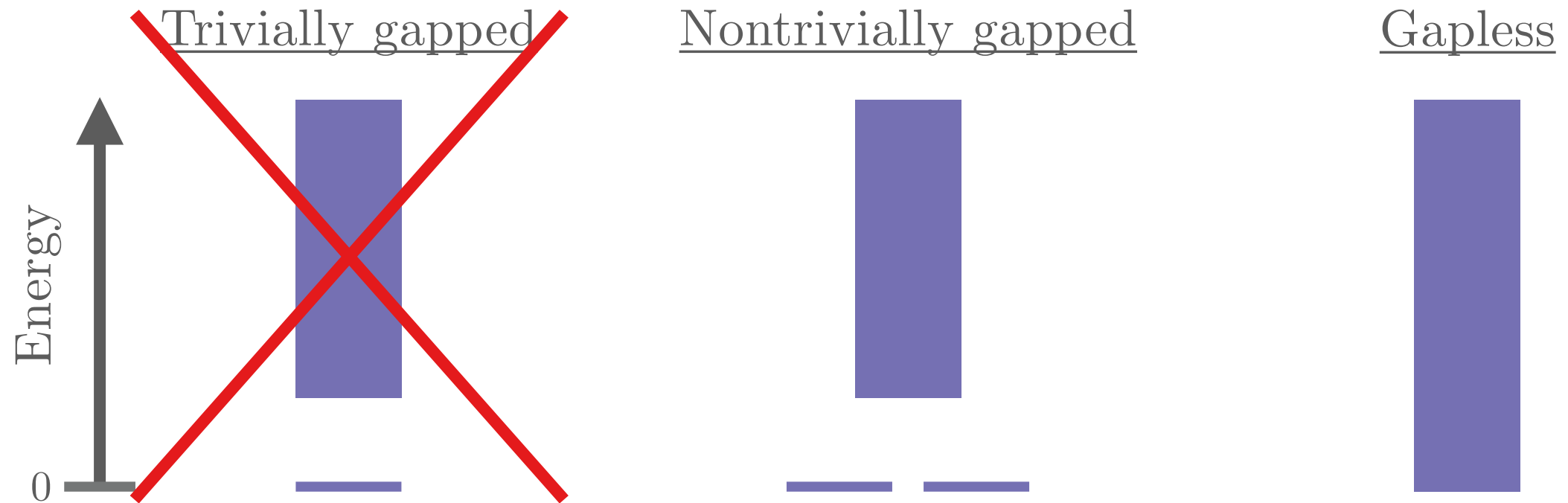
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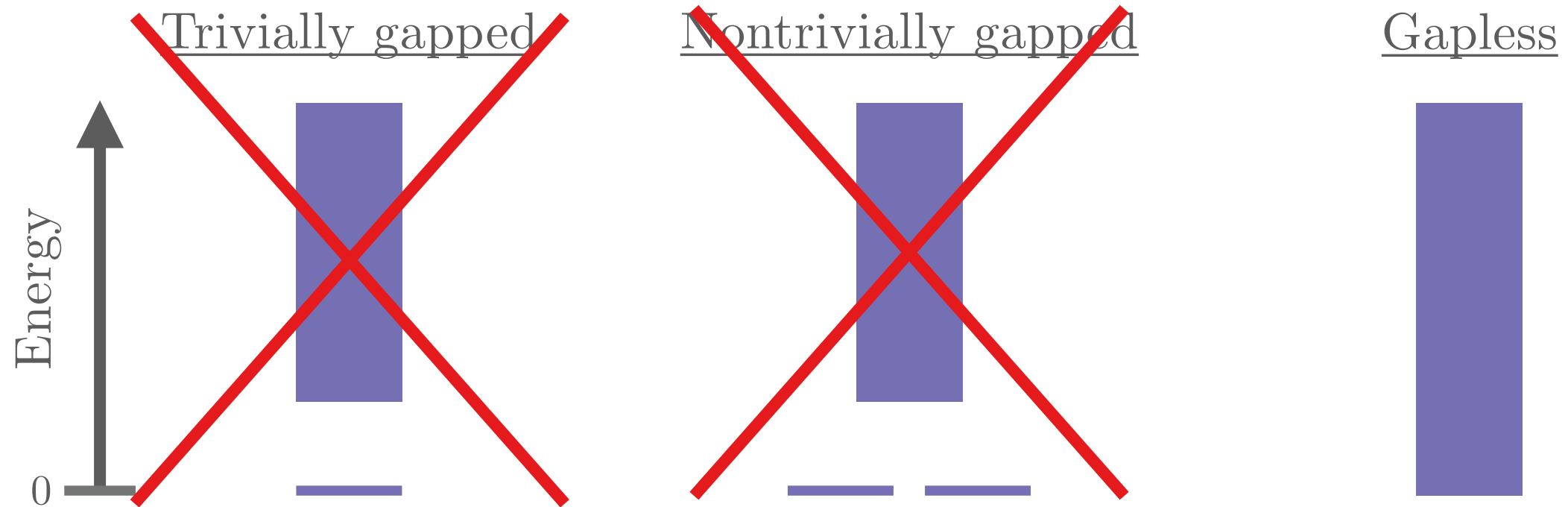
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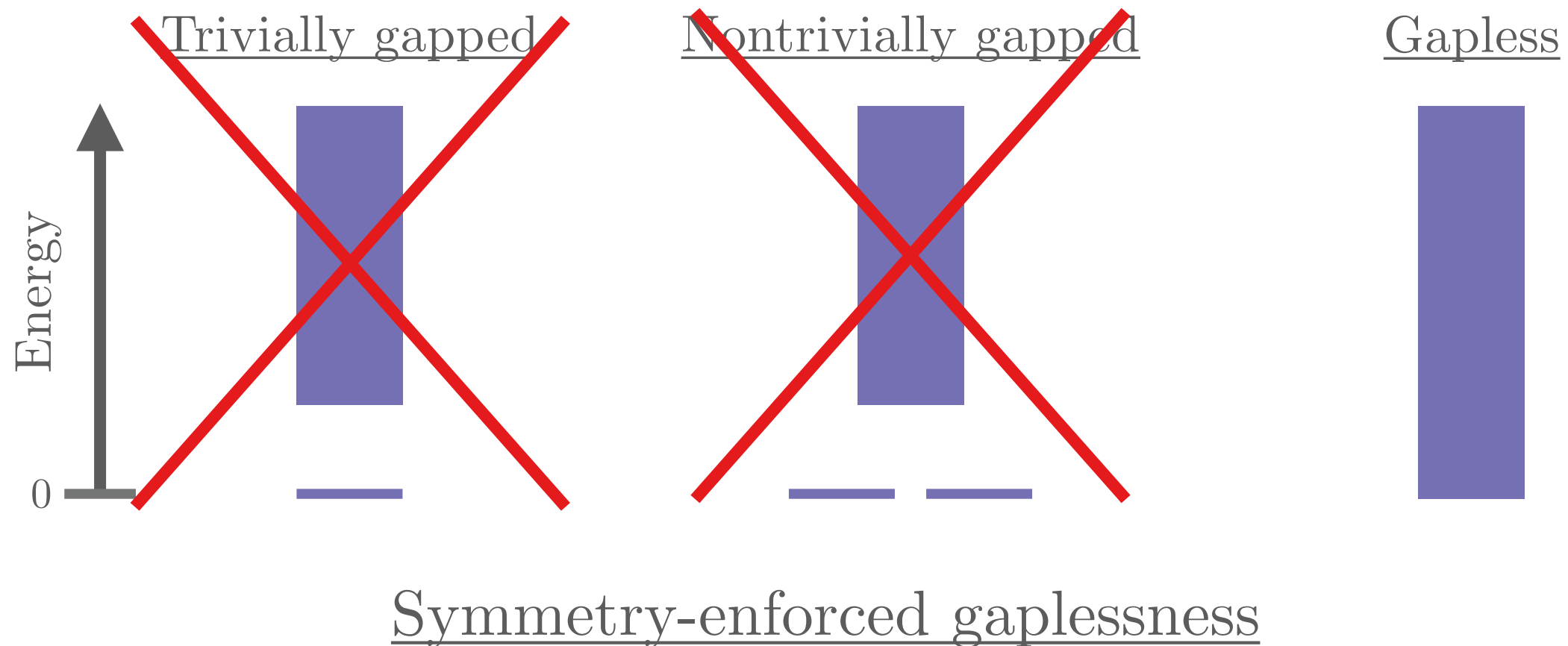
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Sometimes, *symmetry* is even more *powerful*



Condensed Matter



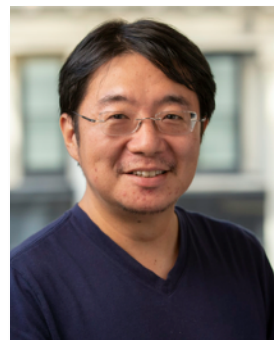
Metlitski



Senthil



Wang



Xu

High Energy



Córdova



Lam



Ohmori



Tachikawa

Two Types of Gaplessness

Not all gaplessness is equal

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Not all gaplessness is equal

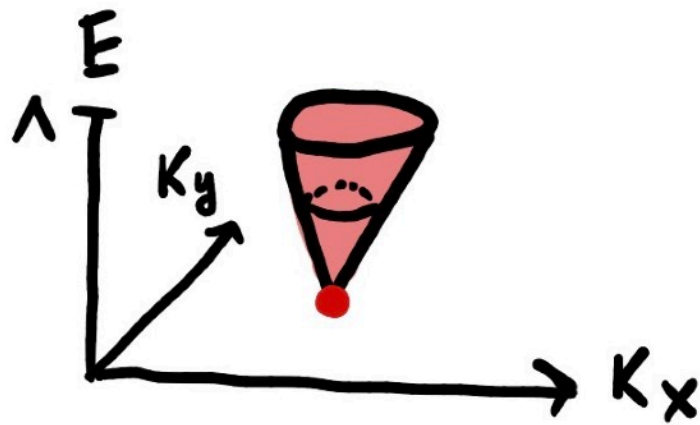
Finite gaplessness

Infinite gaplessness

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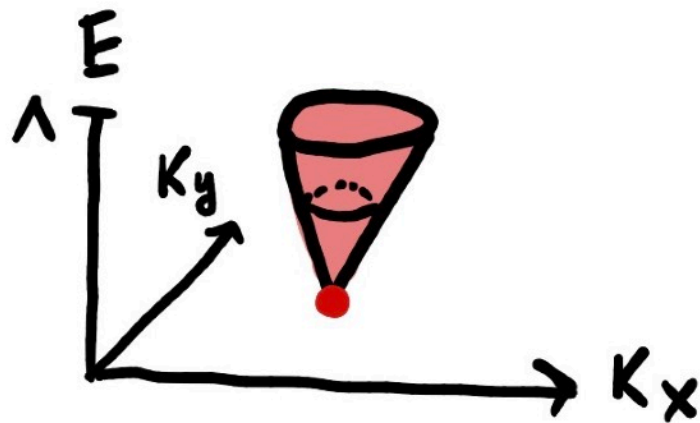


Infinite gaplessness

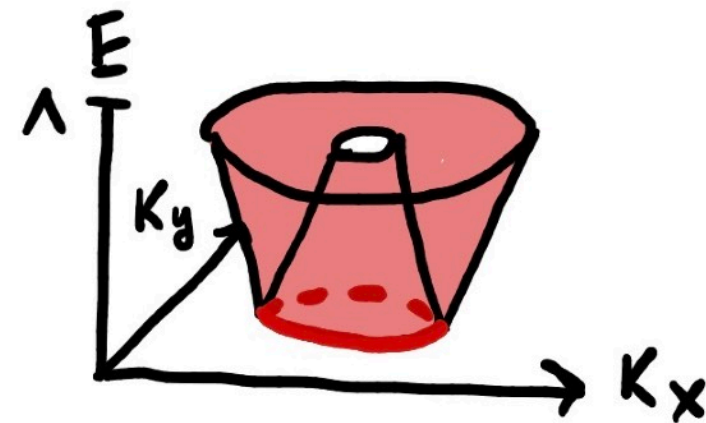
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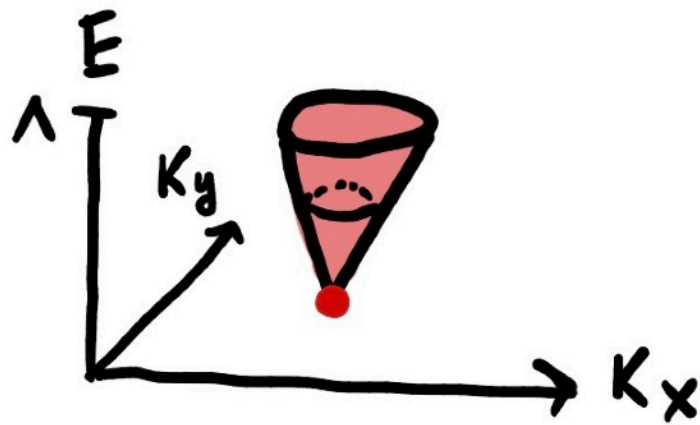
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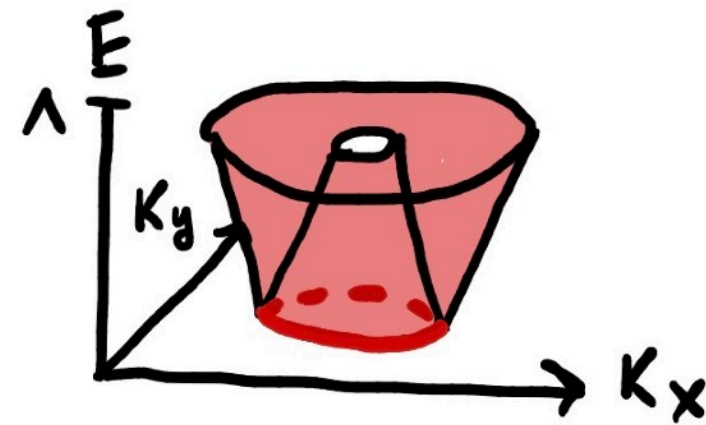
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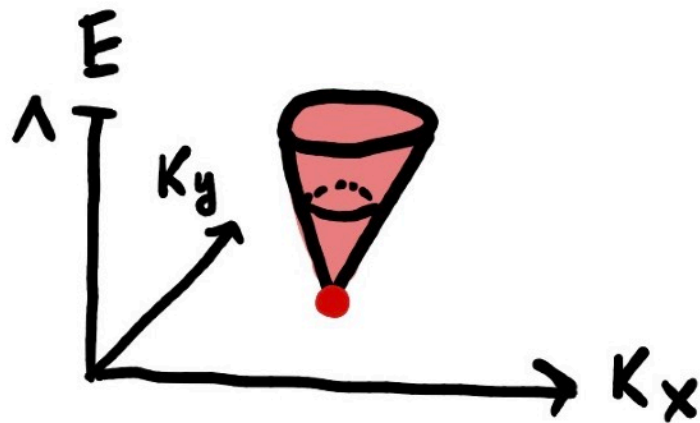


Question: Can a symmetry enforce infinite gaplessness?

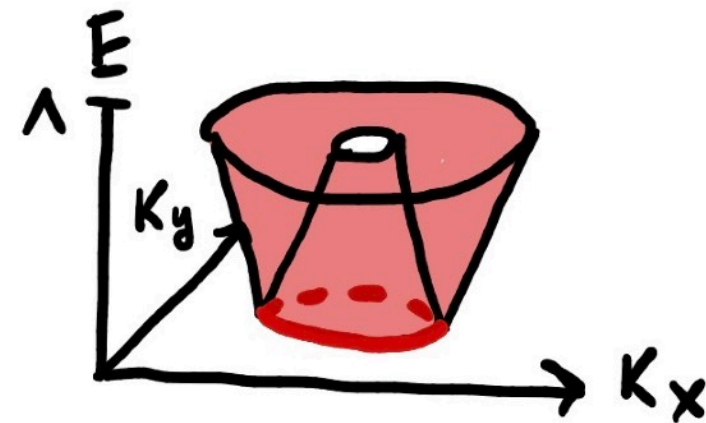
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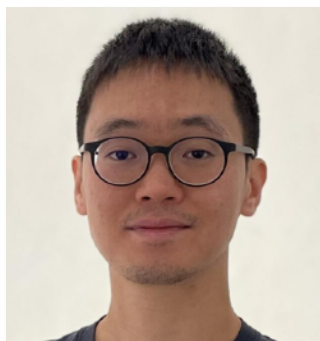


Infinite gaplessness



Question: Can a symmetry enforce infinite gaplessness?

Answer: Yes, symmetry-enforced Fermi surfaces!



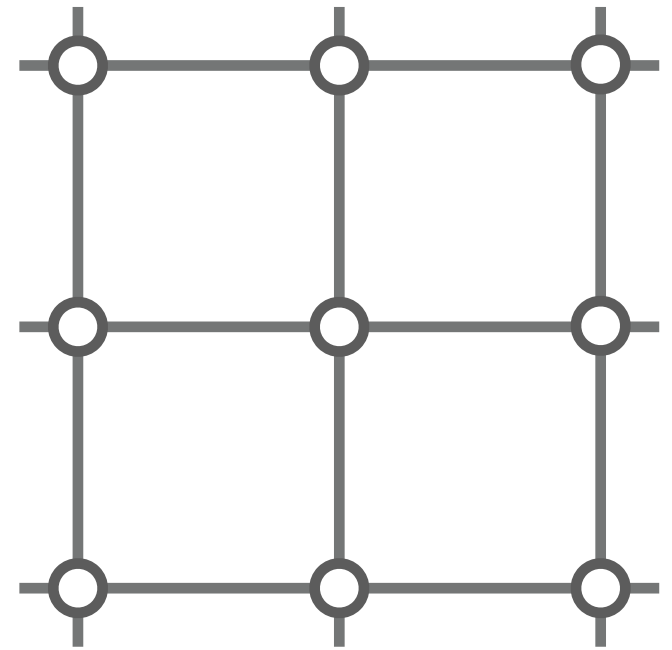
[Kim-SP-Shao, PRL '26]

Symmetry-Enforced Fermi Surface

2D lattice with a **fermion** c_r on each site r

$$\{c_r, c_{r'}^\dagger\} = \delta_{r,r'}$$

$$\{c_r, c_{r'}\} = 0$$



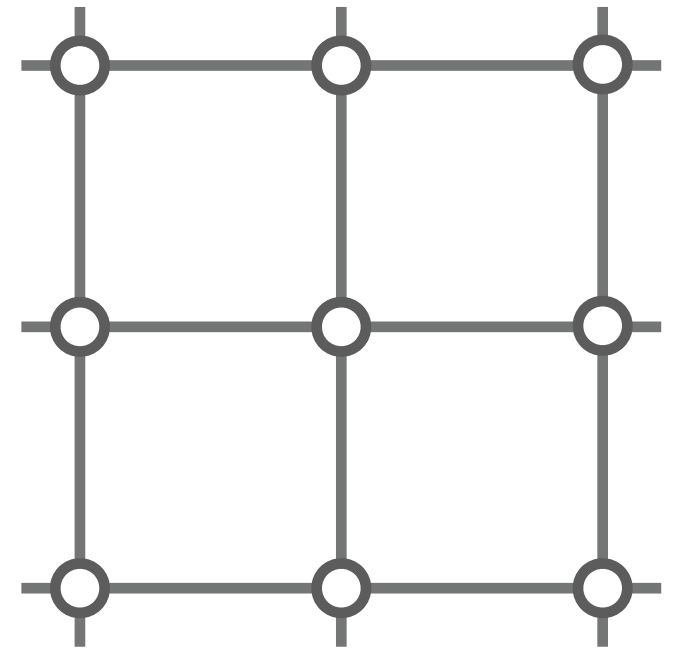
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Be real 😎: $c_r = (a_r + ib_r)/2$

➤ Real (Majorana) **fermions** a_r and b_r



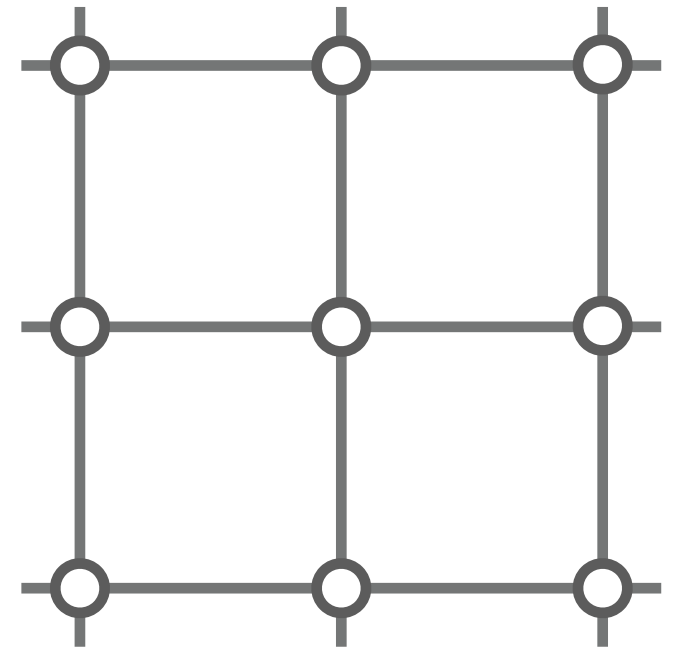
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Symmetry:

1. Fermion number **symmetry**: $c_r \mapsto e^{-i\theta} c_r$
2. Majorana translations: $a_r, b_r \mapsto a_r, b_{r+v}$

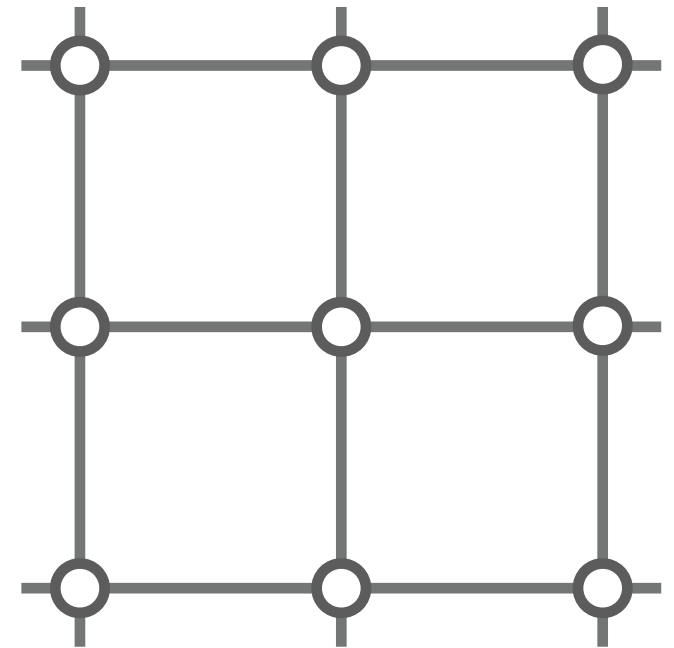
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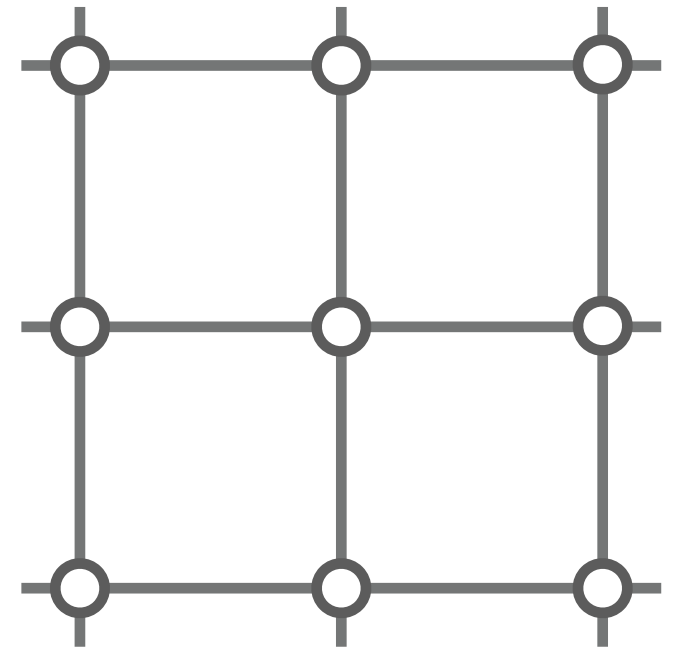
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$$\begin{pmatrix} a_r \\ b_r \end{pmatrix} \mapsto \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} a_r \\ b_r \end{pmatrix}$$

2. Majorana translations: $a_r, b_r \mapsto a_r, b_{r+v}$

$$c_r \mapsto \frac{1}{2}(c_r^\dagger + c_r - c_{r+v}^\dagger + c_{r+v})$$

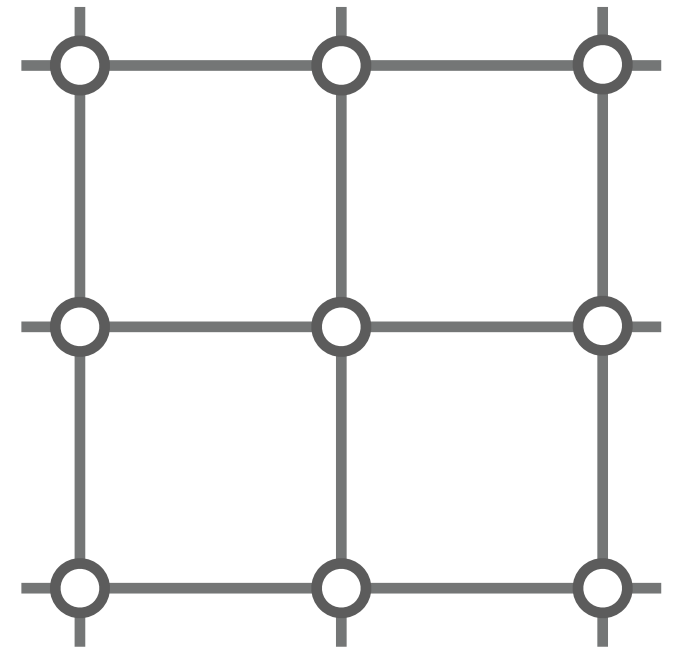
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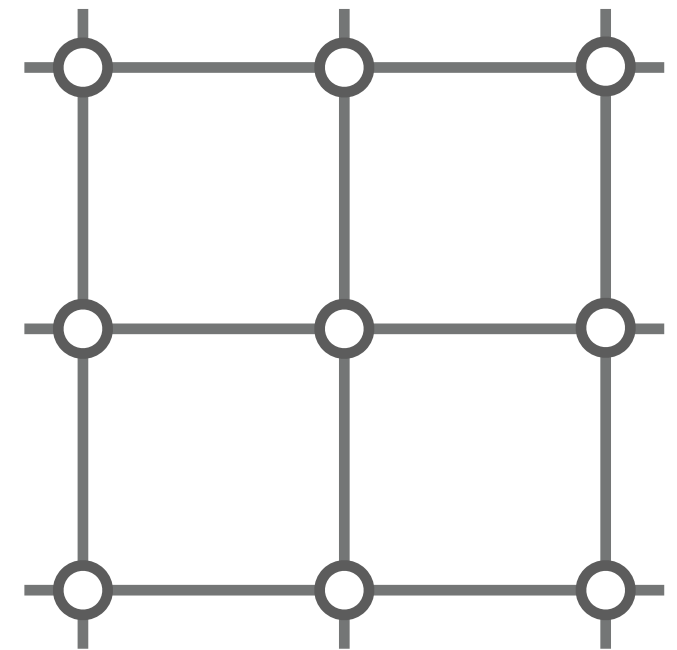
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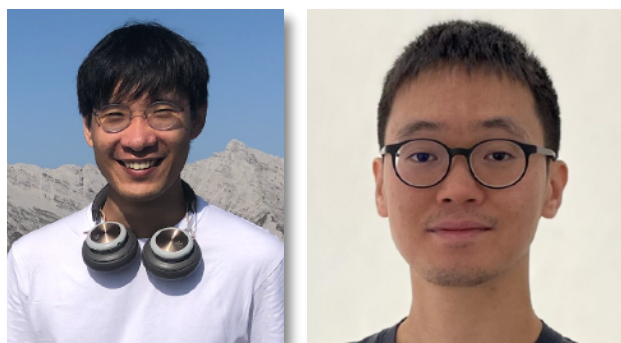


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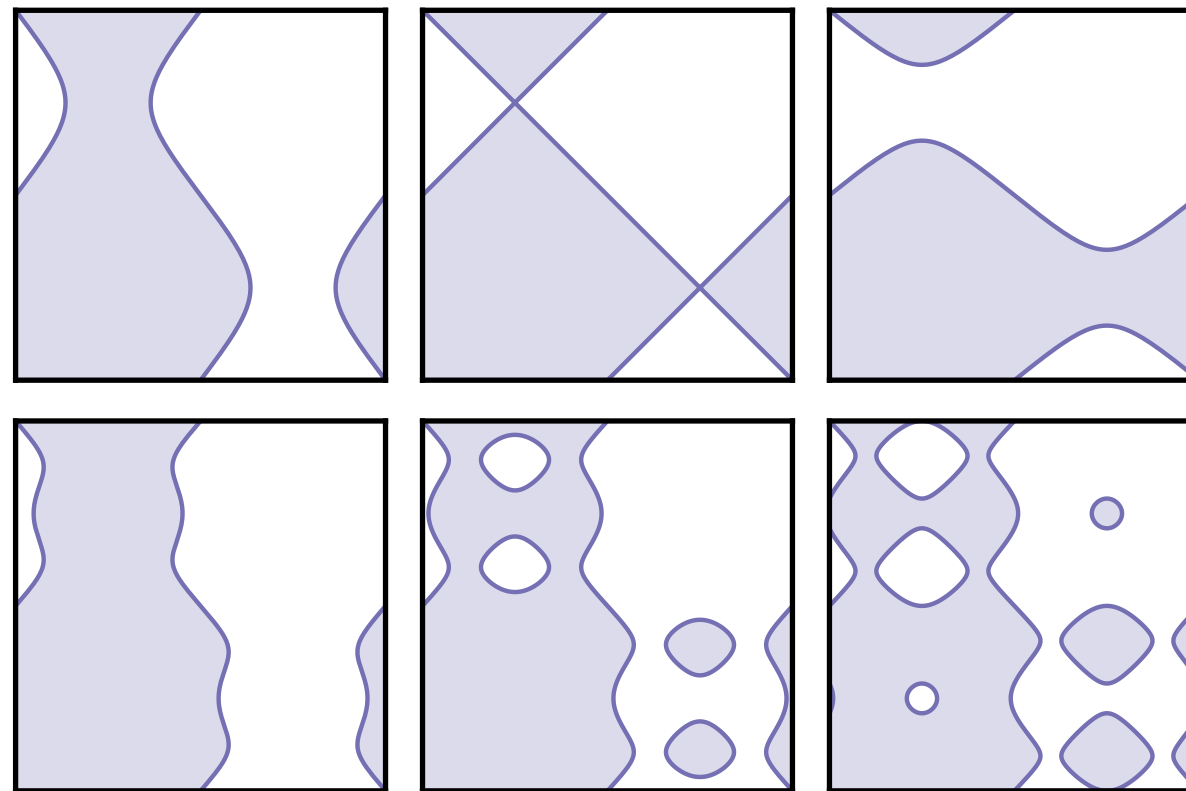
➤ Every **symmetric** Hamiltonian has a **Fermi surface**



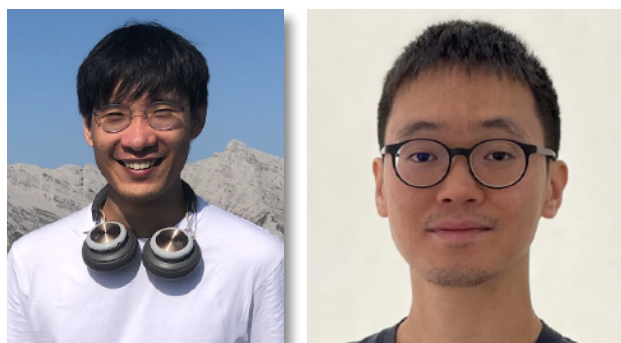
[Kim-**SP**-Shao, PRL '26]

Symmetry-Enforced Fermi Surface

These symmetry-enforced Fermi surfaces are topologically nontrivial: exhibit non-contractible components



➤ Every symmetric Hamiltonian has a Fermi surface



[Kim-SP-Shao, PRL '26]

Onsager Algebra

Fermion number and Majorana translation **symmetry** give rise to conserved operators

$$Q_v = \frac{i}{2} \sum_r a_r b_{r+v} \qquad G_v = \frac{i}{2} \sum_r (a_r a_{r+v} - b_r b_{r+v})$$

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► Form the **Onsager algebra**



1944

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$$[Q_v, Q_{v'}] = iG_{v'-v}$$

$$[G_v, G_{v'}] = 0$$

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Onsager algebra

Quantum Integrability

Math

Symmetry

Gifts from Onsager

Constraint

Symmetry-enforced Fermi surfaces

[Kim-**SP**-Shao, PRL '26]

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Symmetry-enforced **Fermi** surfaces

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1+1d **Gaplessness**

[Chatterjee-**SP**-Shao, PRL '25]

SP-Chatterjee-Shao, SciPost '25

Lew-Smith, **SP**, Shao, arXiv '26]

Symmetry-protected **Weyl** fermions

[Gioia-Thorngren, PRL '26]

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't Hooft Anomaly

LU(1) anomaly



Schwinger anomaly



Witten anomaly



Parity anomaly



First Lesson

Symmetry in quantum systems can constrain
the low-energy/long-distance behavior

Quantum Entanglement

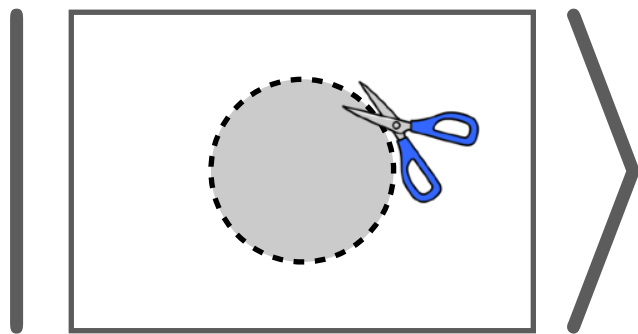
Entanglement Patterns

Given a **quantum system** with many interacting degrees of freedom, what is its ground state's **spatial entanglement pattern**?

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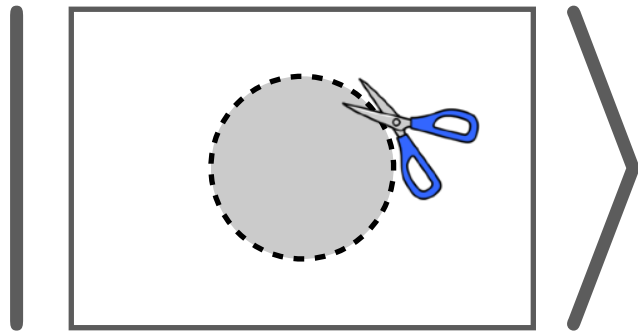
No entanglement



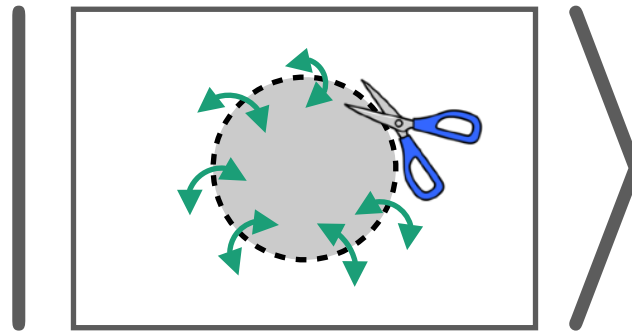
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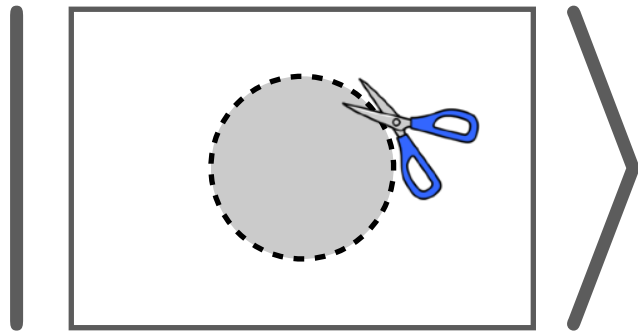
Short-range entanglement



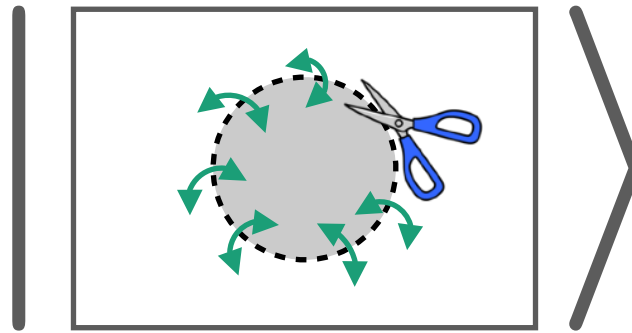
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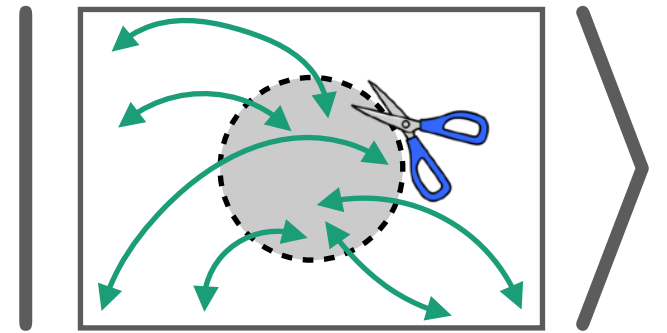
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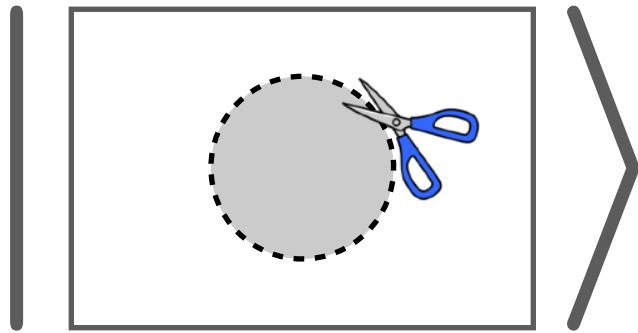
Long-range entanglement



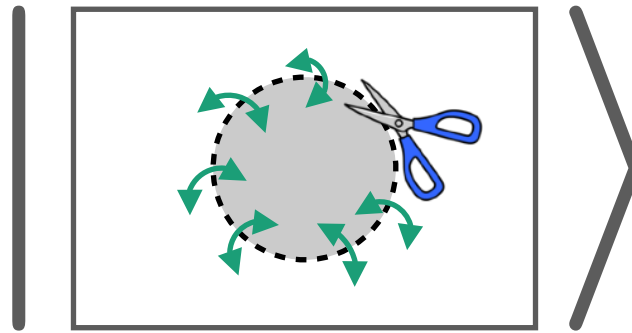
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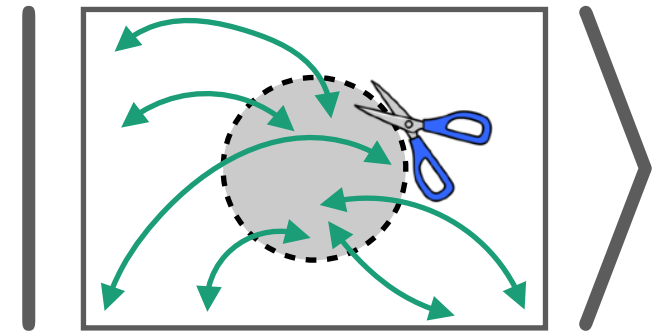
No entanglement



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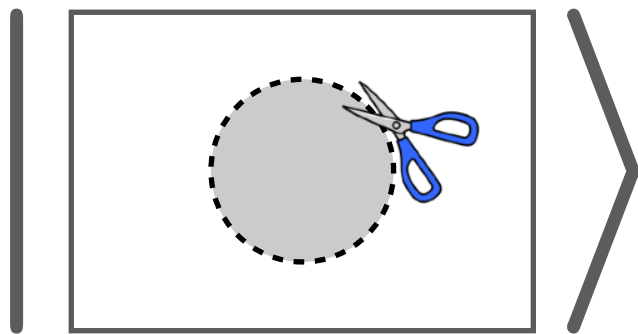


- Notoriously difficult: **Exponential complexity** of the many body Hilbert space

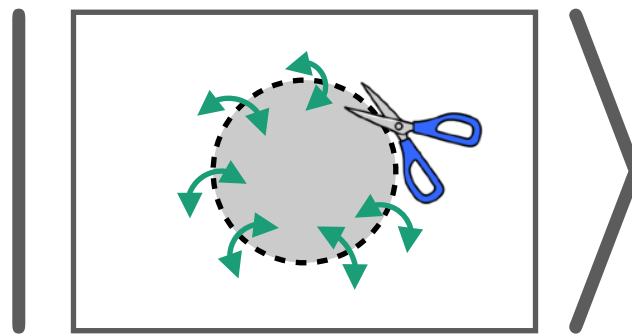
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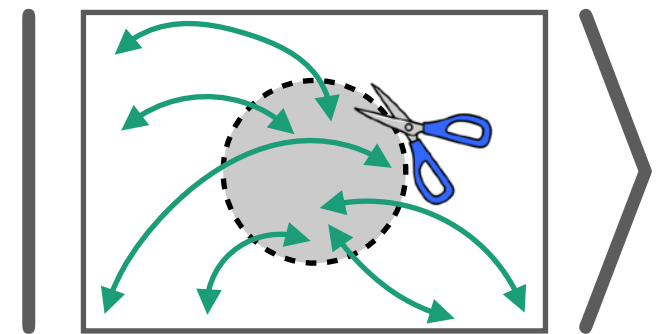
No entanglement



Short-range entanglement



Long-range entanglement



- Notoriously difficult: **Exponential complexity** of the many body Hilbert space

Symmetry can constrain the **ground state entanglement** and organize distinct possibilities

Symmetry-Enforced Entanglement.....

Symmetry can enforce states to have nontrivial entanglement

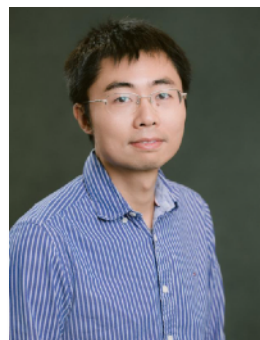
Interface of Condensed Matter and Quantum Information



Cheng



Else



Lu



Oshikawa



Ran



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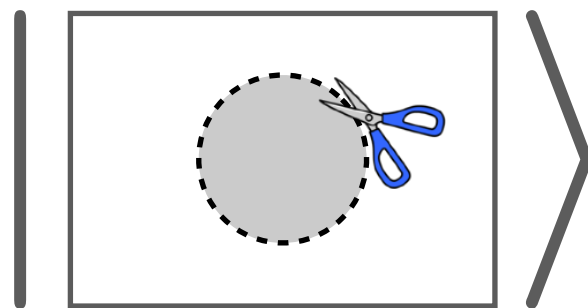


Vishwanath

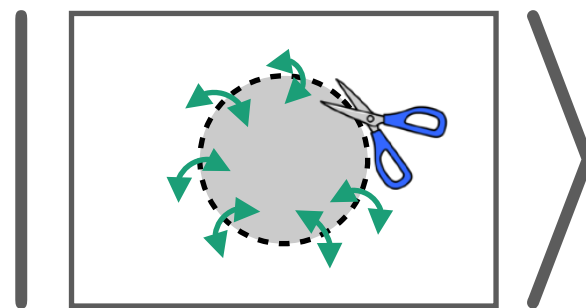
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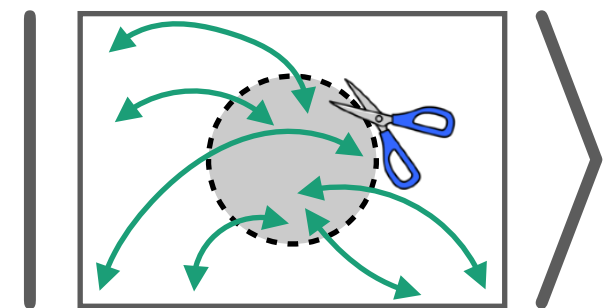
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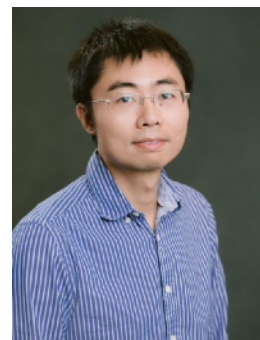
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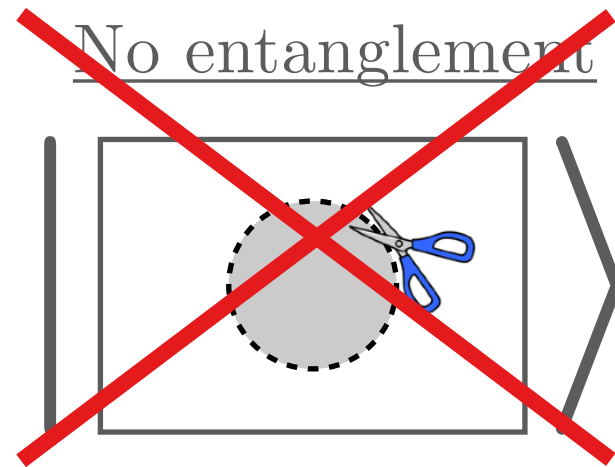
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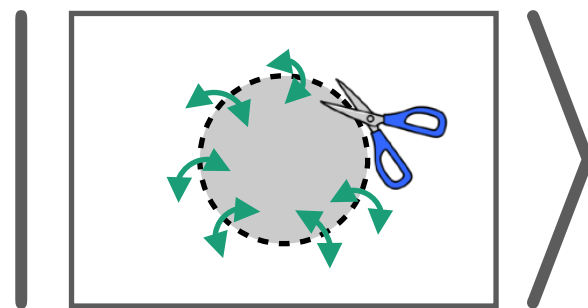
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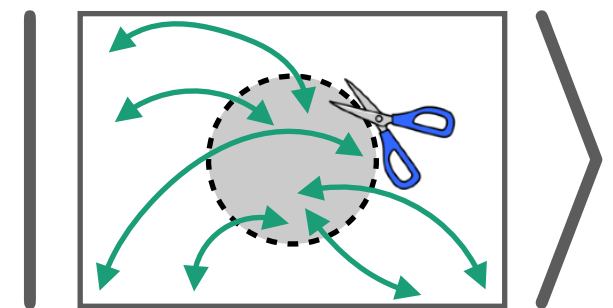
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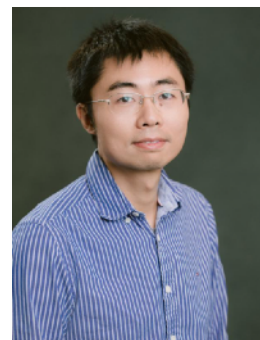
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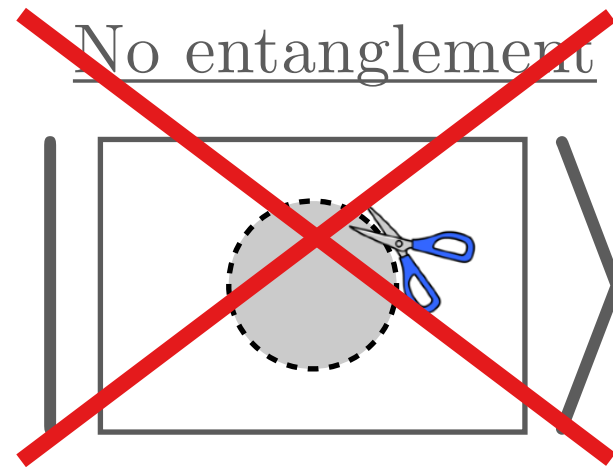
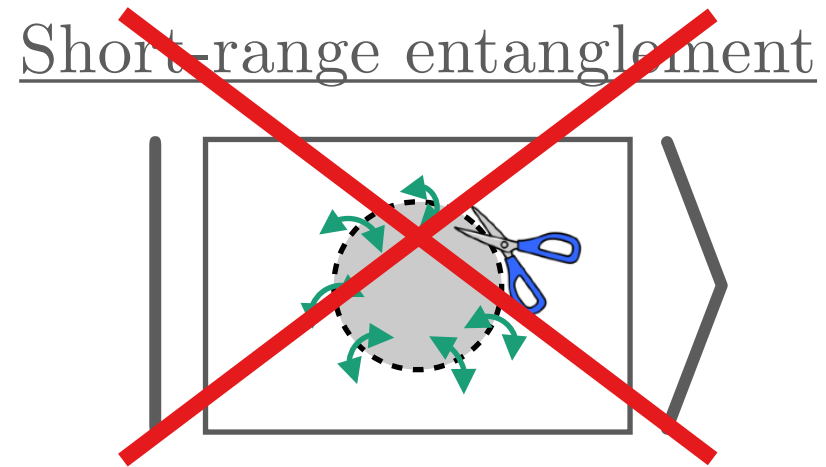
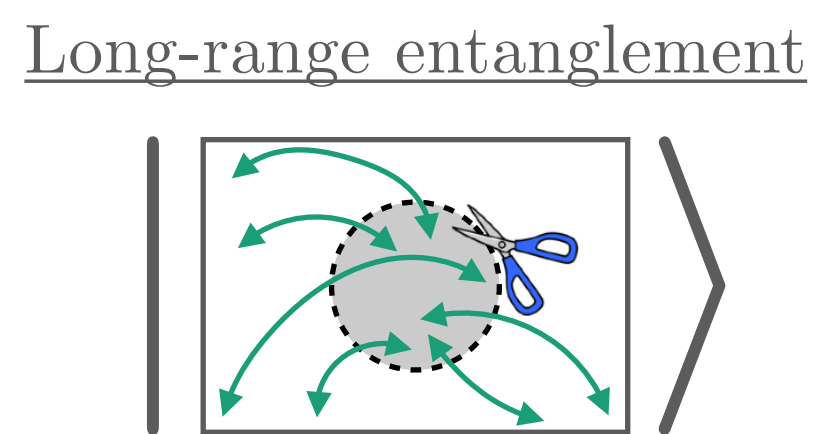
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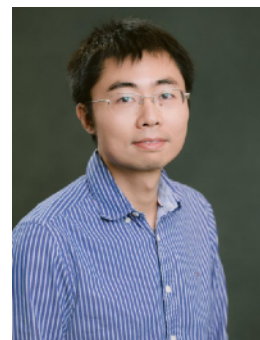
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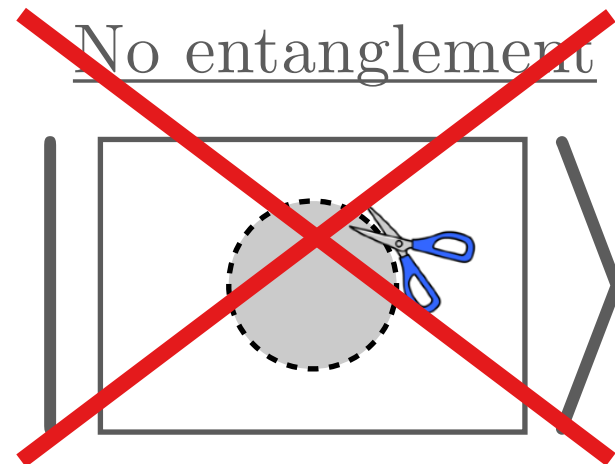
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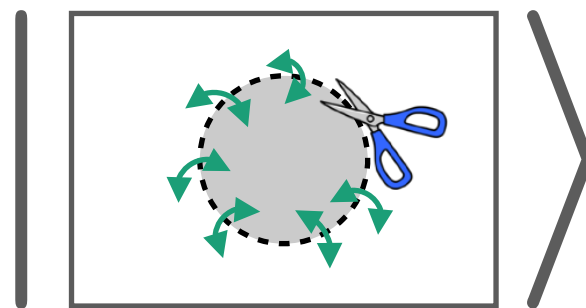
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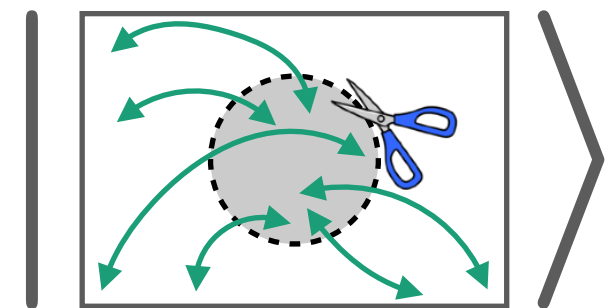
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Short-range entanglement



Long-range entanglement



Example: Locally projective symmetry



$$U = \prod_j U_j$$

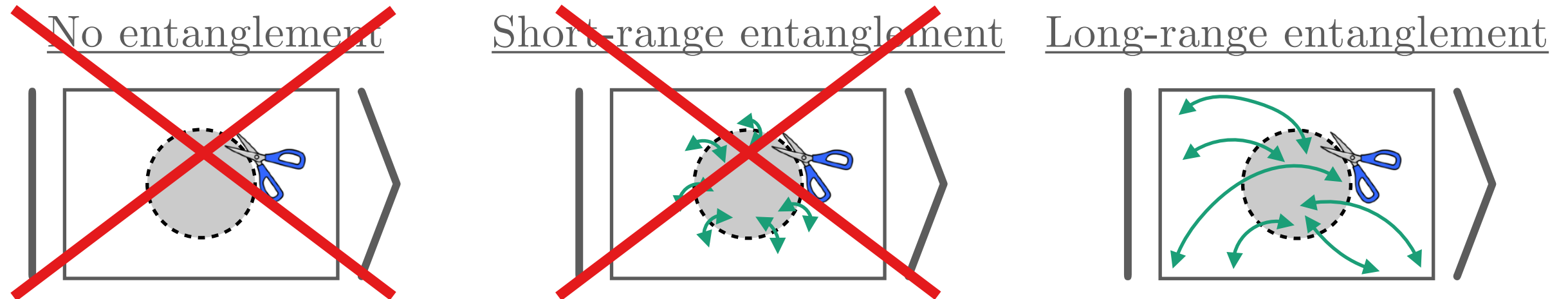
► Trivially entangled state: $|\psi\rangle = \bigotimes_j |\psi_j\rangle$

$U |\psi\rangle = |\psi\rangle \implies U_j$ acts on $|\psi_j\rangle$ as a c-number:

Impossible for projective U_j

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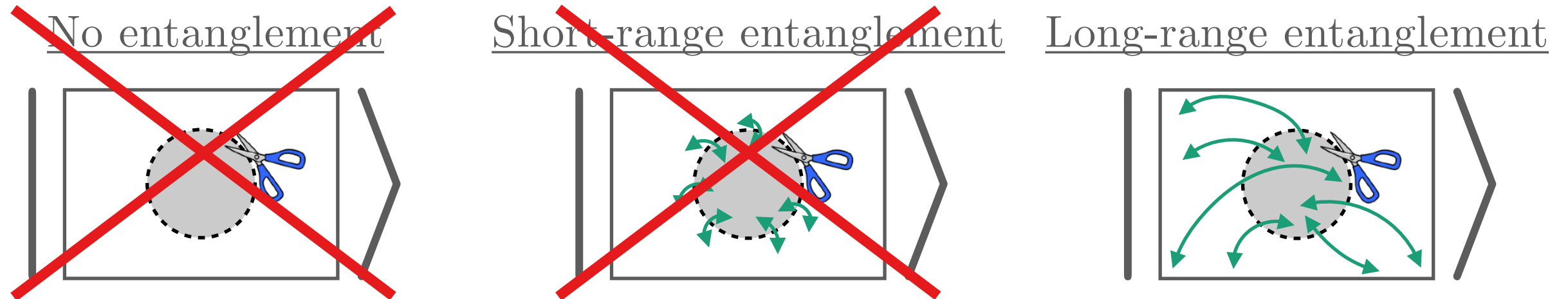


Example: The **LSM theorem**, $SO(3)$ spin rotation and lattice translation symmetry

Spin	Trivially gapped phases?	Symmetry-enforced long-range entanglement?
Half-integer	None	Yes
Integer	Exist	No

Symmetry-Enforced Entanglement

Symmetry can enforce states to have nontrivial entanglement

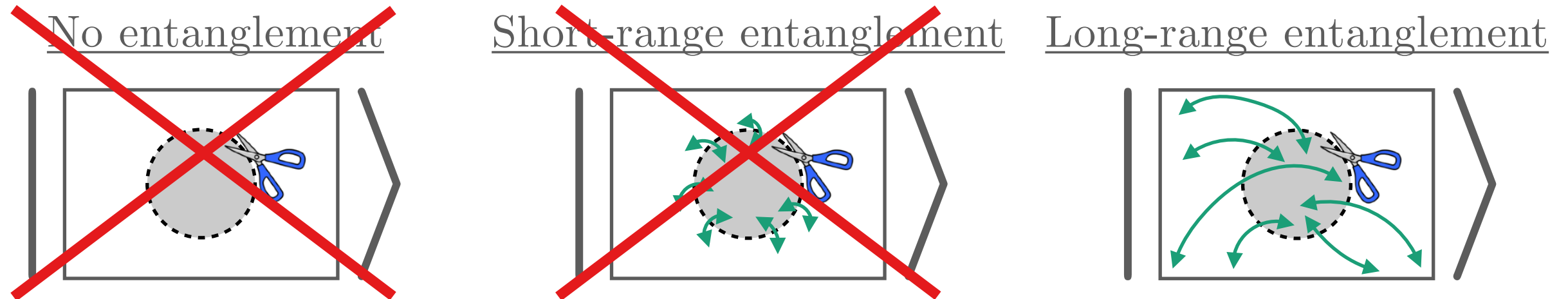


Example: The **LSM theorem**, $\text{SO}(3)$ spin rotation and lattice translation symmetry

► $\vec{S}_j \mapsto R \vec{S}_j$ is a locally projective symmetry for half-integer spin

Symmetry-Enforced Entanglement

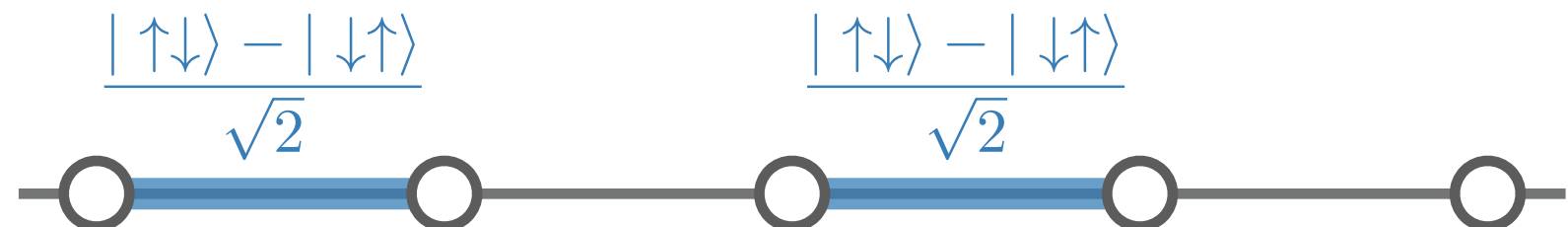
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Example: The **LSM theorem**, $SO(3)$ spin rotation and lattice translation symmetry

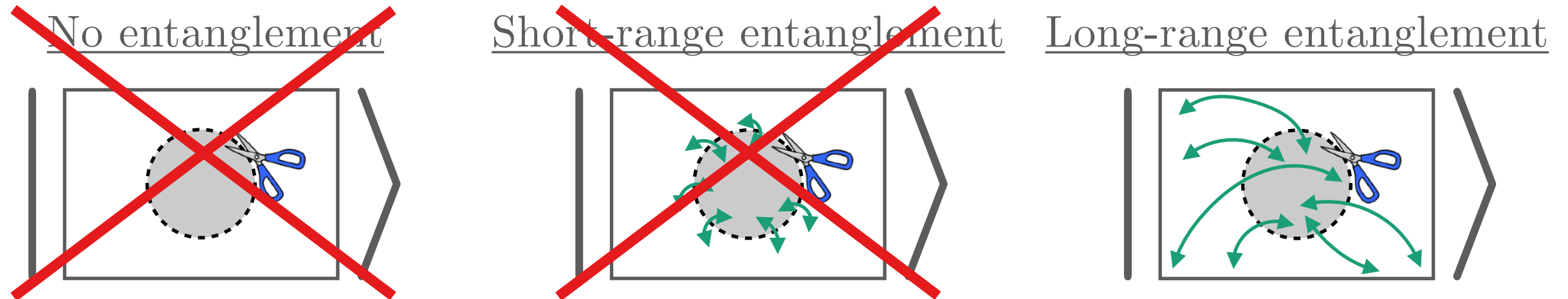
- $\vec{S}_j \mapsto R \vec{S}_j$ is a locally projective symmetry for half-integer spin
 $\implies SO(3)$ symmetric states with short-range entanglement

e.g., spin $1/2$



Symmetry-Enforced Entanglement

Symmetry can enforce states to have nontrivial entanglement



Example: The **LSM theorem**, $SO(3)$ spin rotation and lattice translation symmetry

- $\vec{S}_j \mapsto R \vec{S}_j$ is a locally projective symmetry for half-integer spin
 - $\implies$ $SO(3)$ symmetric states with short-range entanglement
 - $\implies$ None are lattice translation symmetric

Second Lesson

Symmetry in quantum systems can constrain
the ground state entanglement pattern

Generalizing Symmetry

Generalized Symmetries

The last ten years have seen tremendous progress in generalizing the notion of symmetry

Generalized Symmetries

Very **quickly developing** topic! At least 16 reviews:

1. McGreevy, Ann. Rev. Condensed Matter Phys. '22.
2. Córdova, Dumitrescu, Intriligator, Shao, Snowmass '22
3. Freed, Proc. Symp. Pure Math. '22
4. Reece, TASI '23
5. Shao, TASI '23
6. Brennan, Hong, '23
7. Bhardwaj et al., Phys. Rept. '23
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Generalized Symmetries

The last ten years have seen tremendous progress in generalizing the notion of symmetry

- Crucially: Consequences of generalized symmetries are similar to those of ordinary symmetries

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Generalized symmetries can also constrain **low-energy dynamics** and **entanglement patterns**

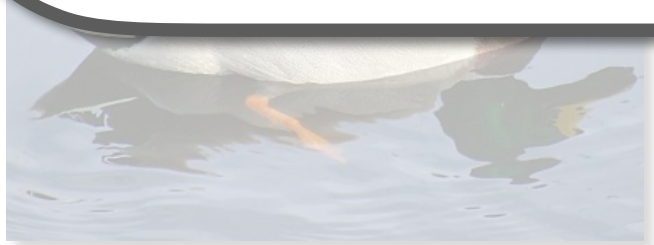
Generalized Symmetries

The last ten years have seen tremendous progress in generalizing the notion of symmetry

➤ Consider a system with a similar

I want to introduce you to two generalized symmetries (there are many more)

1. Modulated symmetries
2. Non-invertible symmetries

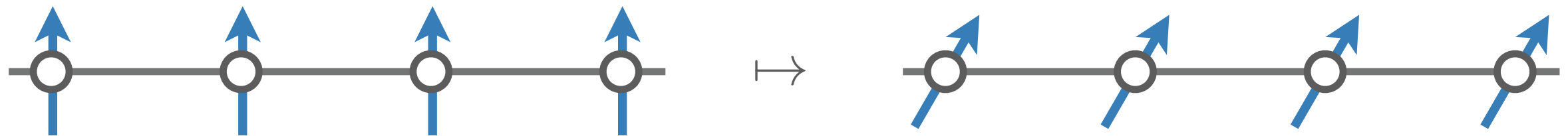


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Generalized symmetries can also constrain low-energy dynamics and entanglement patterns

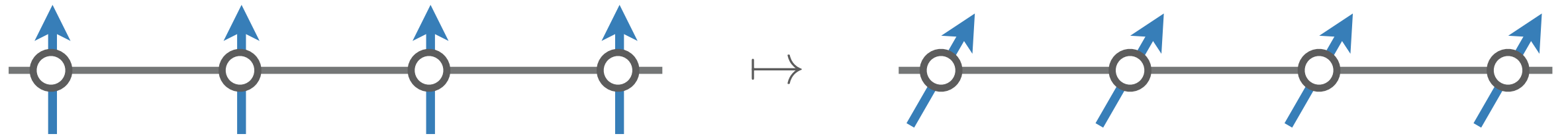
Modulated Symmetry

Ordinary symmetries act uniformly throughout space

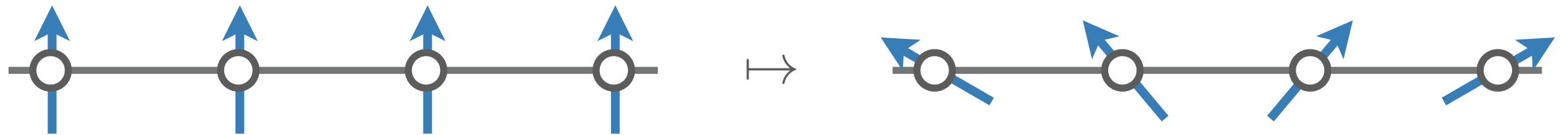


Modulated Symmetry

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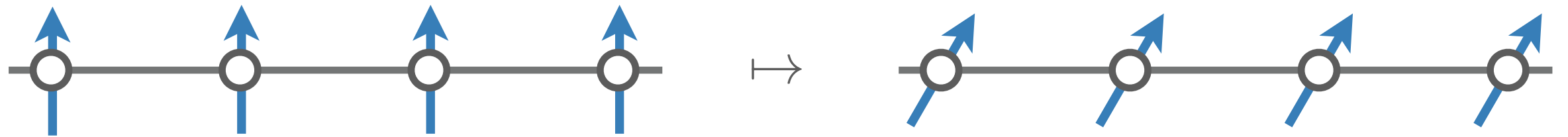
Modulated **symmetries** act in a spatially modulated way



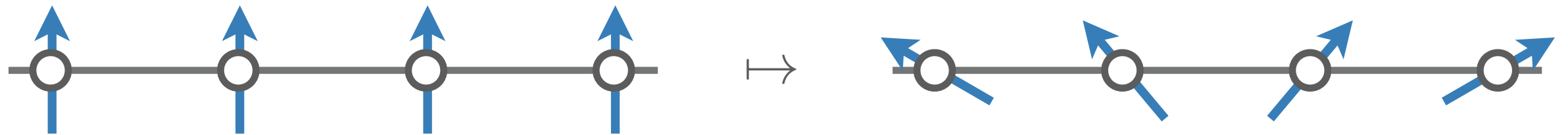
► Ubiquitous in quantum spin models and Lifshitz field theories

Modulated Symmetry

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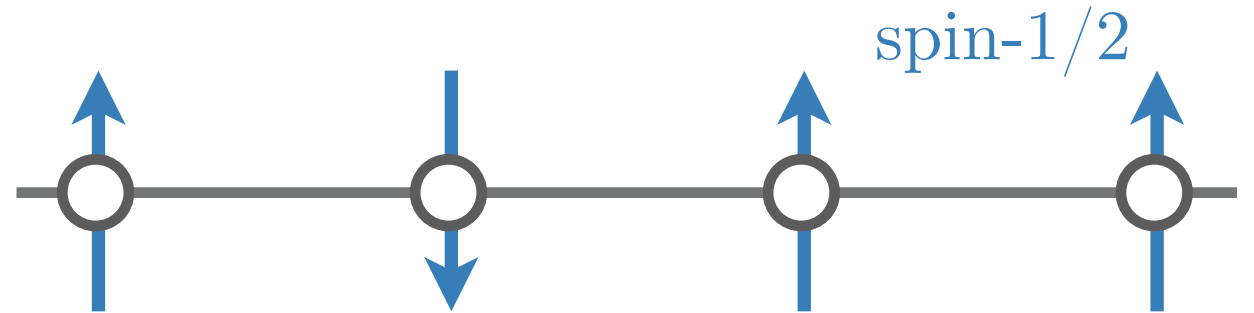
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[**SP**-Bulmash, arXiv '26]

Example: Dipole Symmetry

Consider Quantum spin chain



► Symmetry transformations

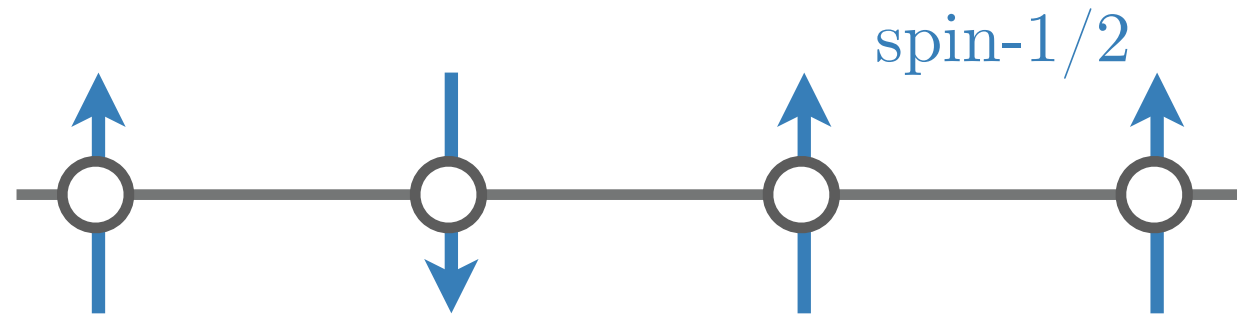
1. $X_j \mapsto (-1)^j X_j$ and $Z_j \mapsto (-1)^{j+1} Z_j$

2. $X_j \mapsto (-1)^{j+1} X_j$ and $Z_j \mapsto (-1)^j Z_j$

$\implies \mathbb{Z}_2 \times \mathbb{Z}_2$ modulated symmetry

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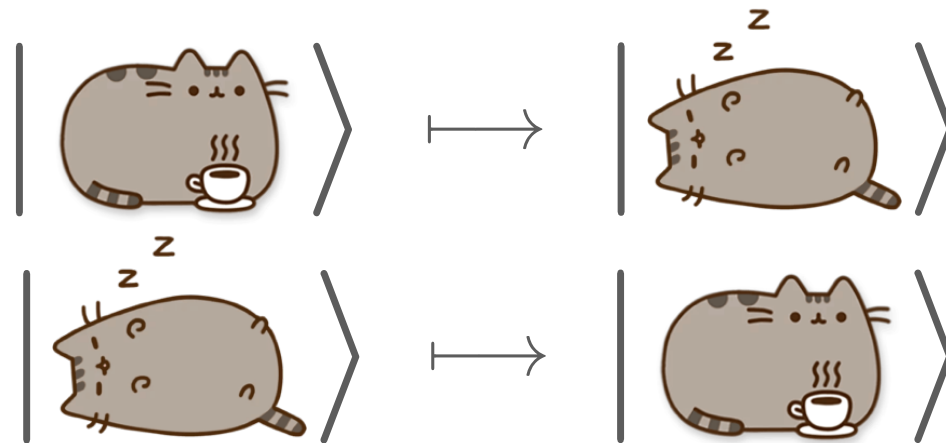
2. $X_j \mapsto (-1)^{j+1} X_j$ and $Z_j \mapsto (-1)^j Z_j$

$\implies \mathbb{Z}_2 \times \mathbb{Z}_2$ modulated symmetry

Total symmetry	Trivially gapped phases?	Symmetry -enforced long range entanglement ?
Modulated symmetry	Exist	No
With translation symmetry	None	Yes

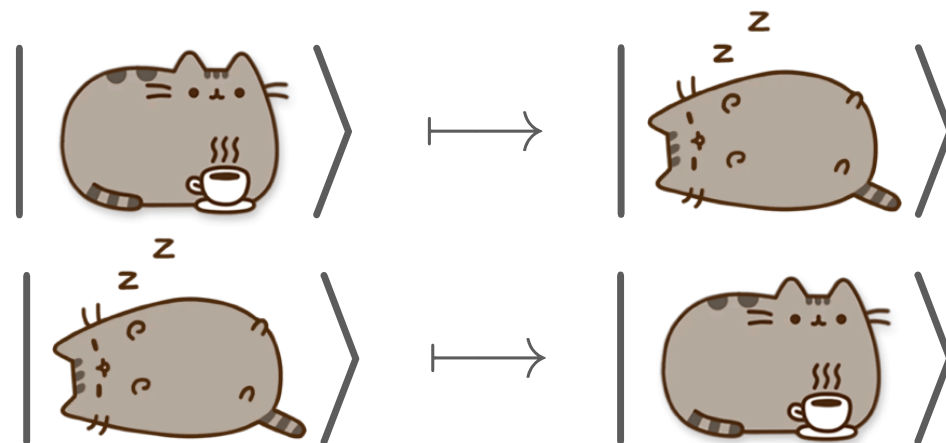
Non-Invertible Symmetry

Ordinary *symmetry* transformations have inverses

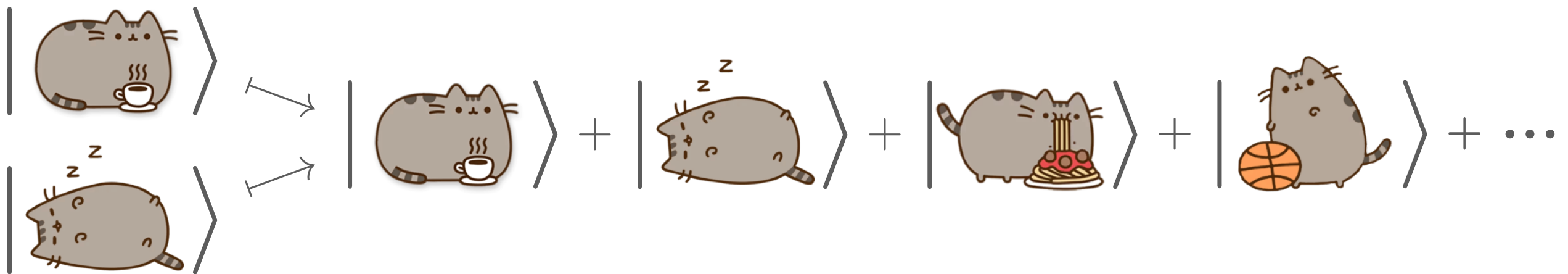


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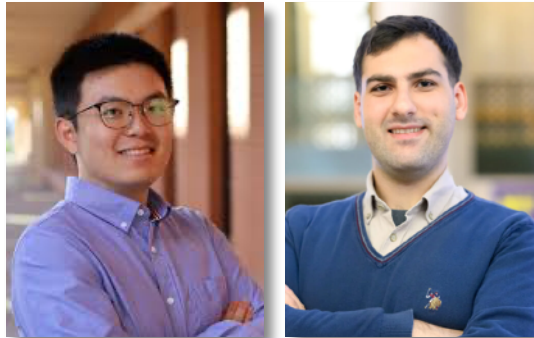
Non-invertible **symmetry** transformations have no inverse!



➤ Arise in simple quantum lattice systems and field theories

Non-Invertible Symmetry

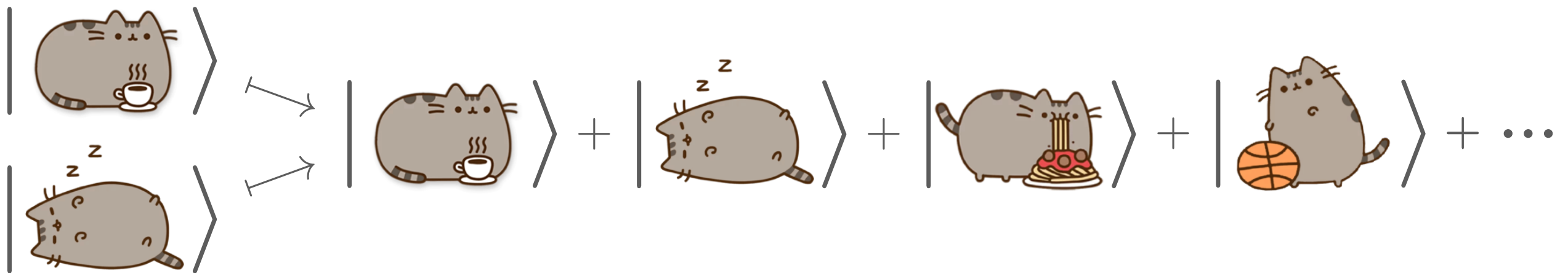
Non-invertible symmetries can constrain low-energy dynamics and entanglement patterns



[SP-Lam-Aksoy, SciPost Phys. '25]

[SP-Aksoy-Lam, SciPost Phys. '26]

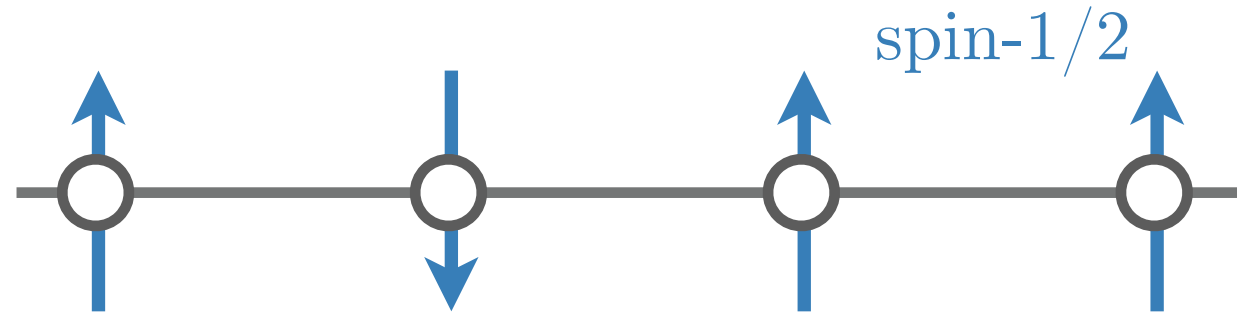
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Example: Kramers-Wannier Symmetry

Consider Quantum spin chain



► Symmetry transformation

$$X_j \mapsto Z_j Z_{j+1} \qquad Z_j Z_{j+1} \mapsto X_{j+1}$$

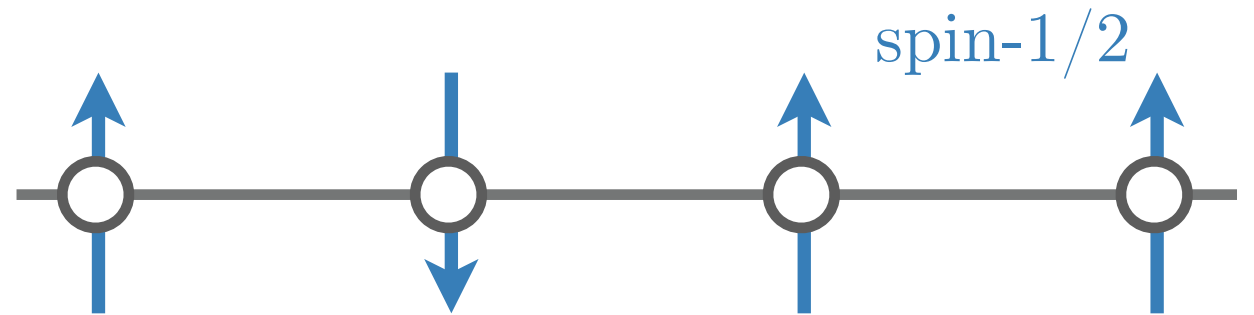
$\Rightarrow$ Non-invertible transformation

$$1 \mapsto 1 \quad \text{and} \quad \prod_j X_j \mapsto \prod_j Z_j Z_{j+1} = 1$$

$\Rightarrow$ Symmetry of the Ising model $H = - \sum_j (Z_j Z_{j+1} + X_j)$

Example: Kramers-Wannier Symmetry

Consider **Quantum spin chain**



➤ **Symmetry** transformation

$$X_j \mapsto Z_j Z_{j+1} \quad Z_j Z_{j+1} \mapsto X_{j+1}$$

⇒ Non-invertible transformation

$$1 \mapsto 1 \quad \text{and} \quad \prod_j X_j \mapsto \prod_j Z_j Z_{j+1} = 1$$

⇒ Symmetry of the Ising model $H = - \sum_j (Z_j Z_{j+1} + X_j)$

➤ Forbids trivially gapped phases and enforces **long-range entanglement**.



Seiberg



Seifnashri



Shao

Third Lesson

Generalized symmetries also have powerful,
unexpected consequences!

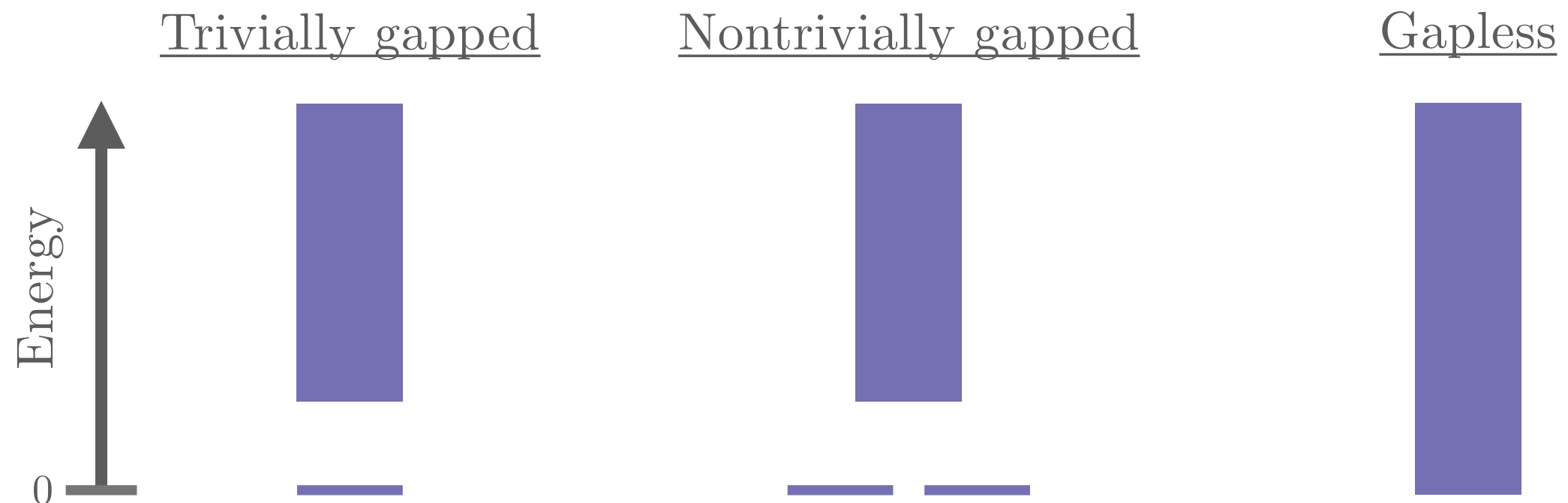
Summary

Symmetry in quantum physics has unexpected consequences

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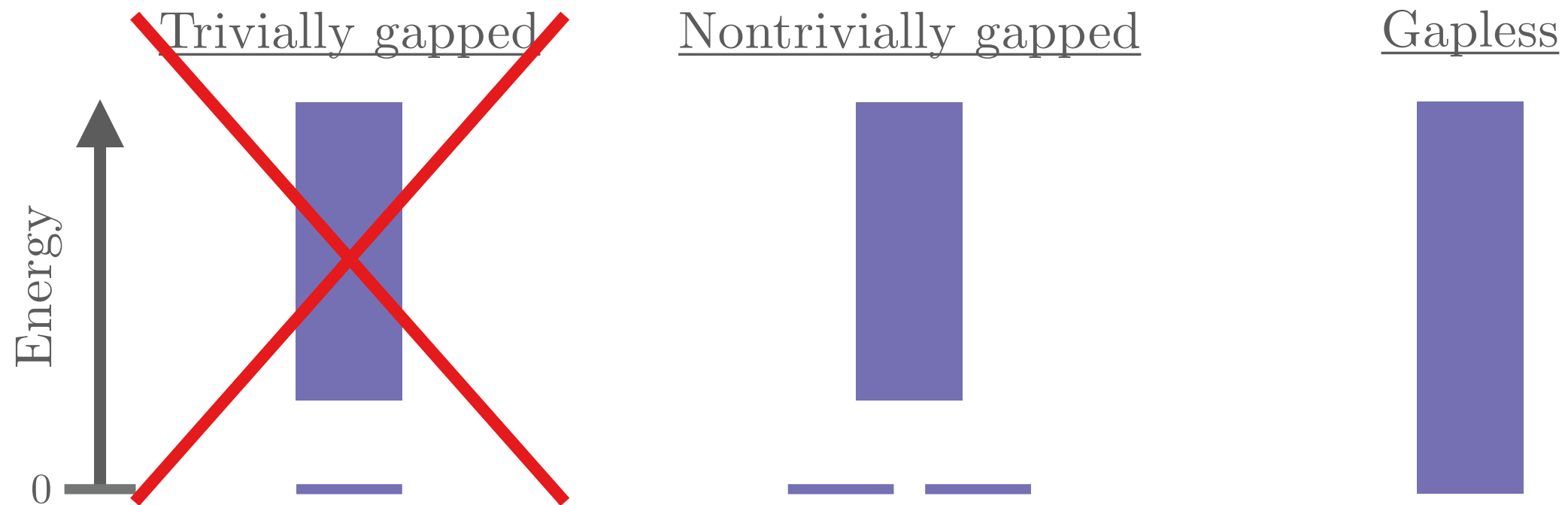
1. Constrains low-energy dynamics



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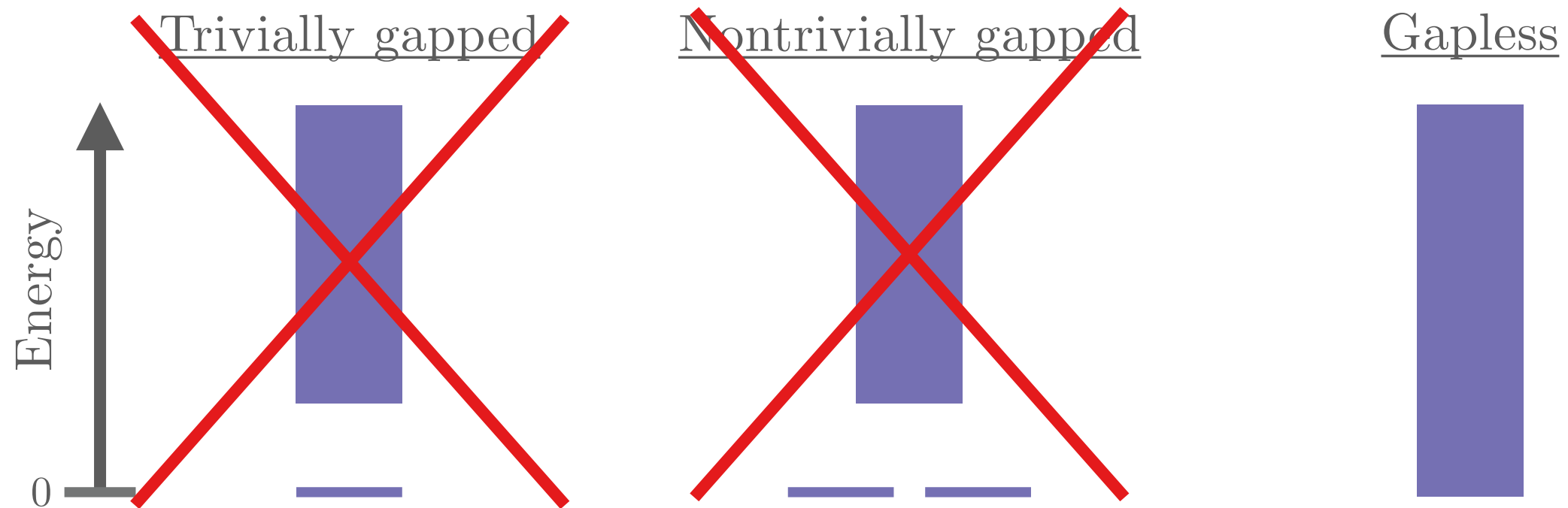
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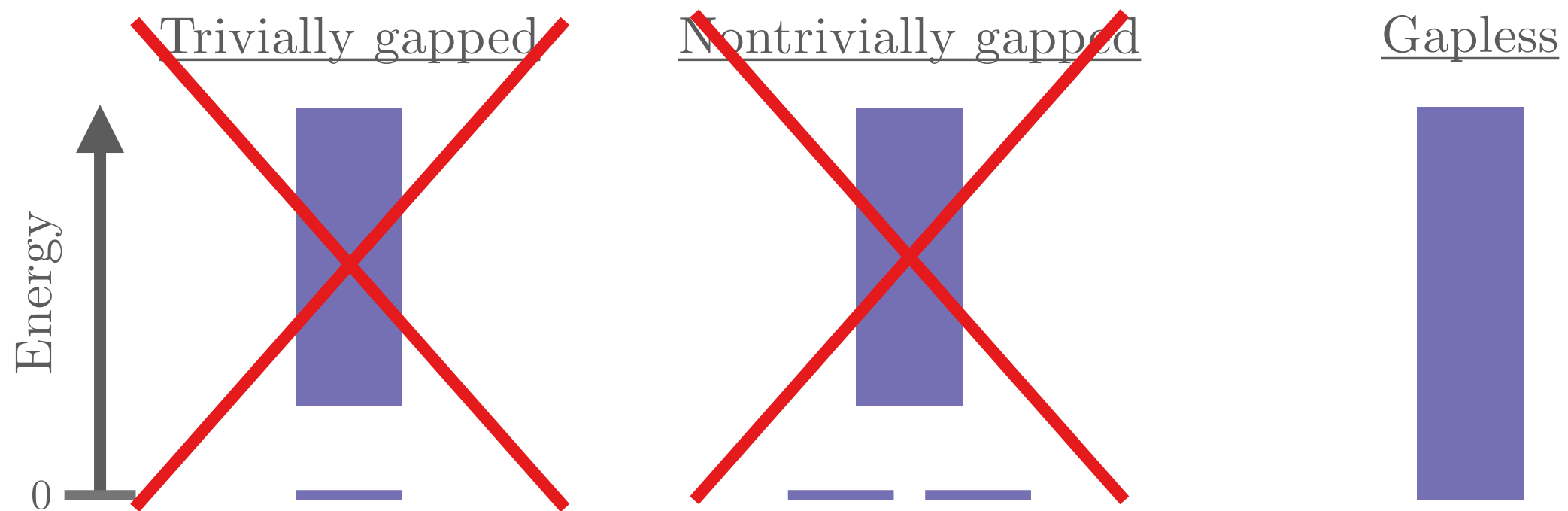
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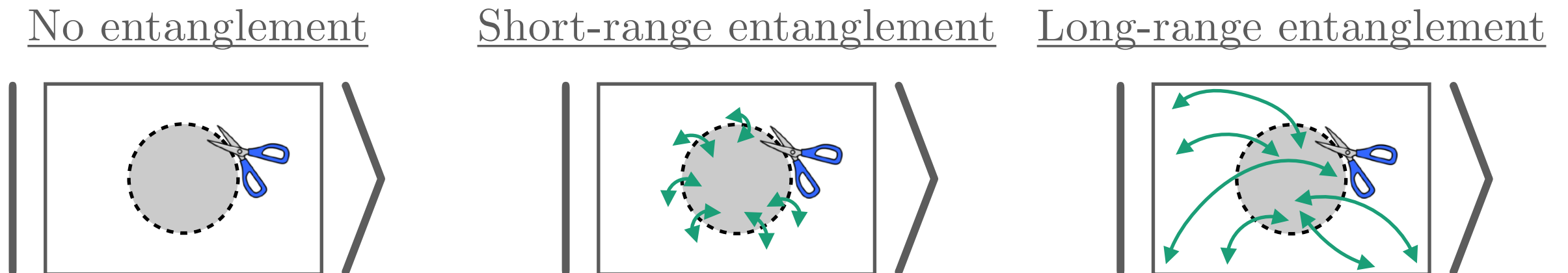
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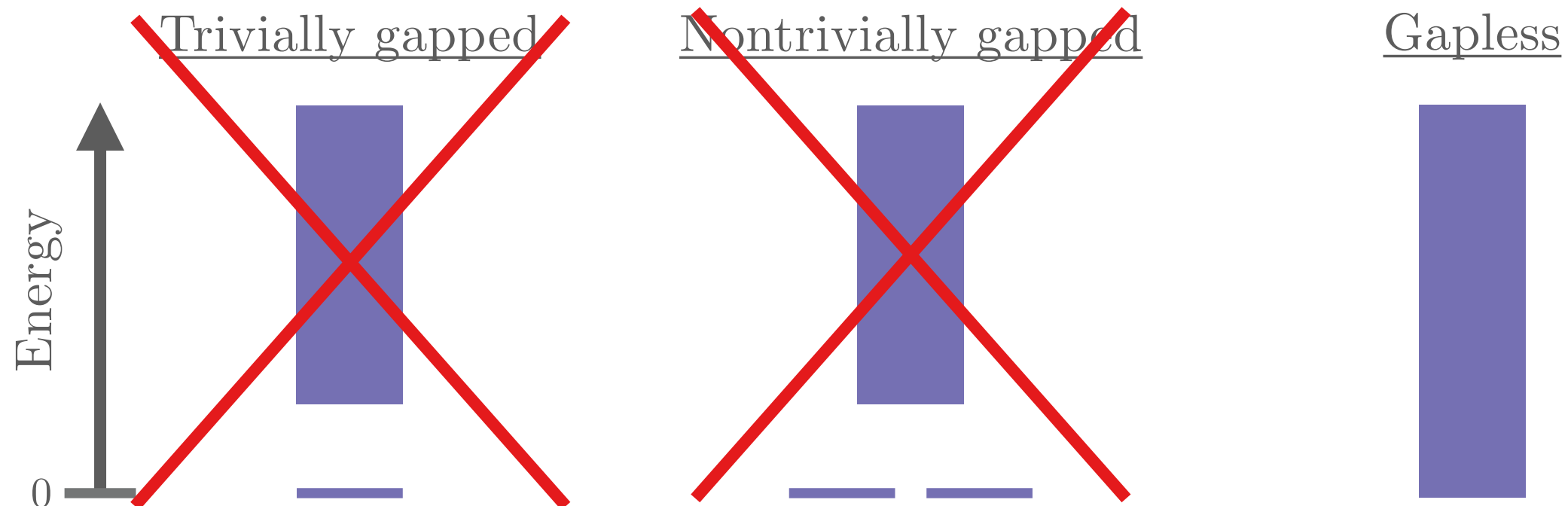
2. Constrains ground state entanglement



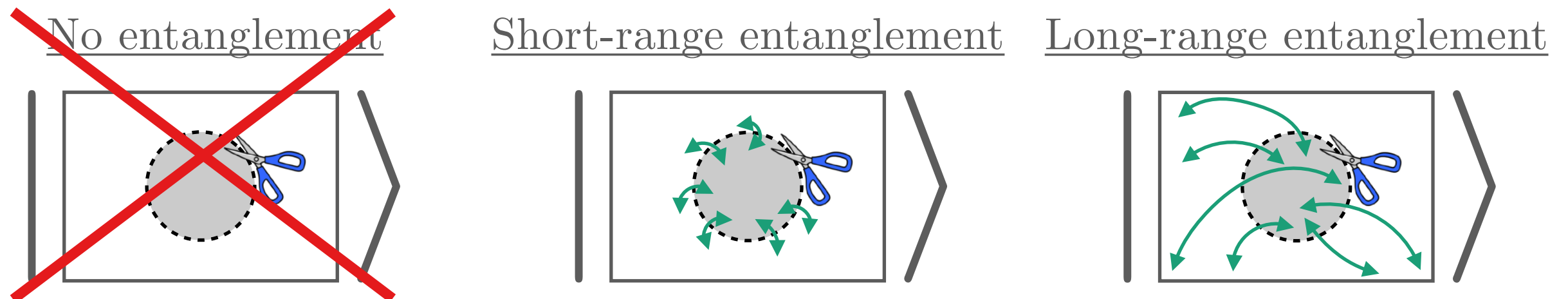
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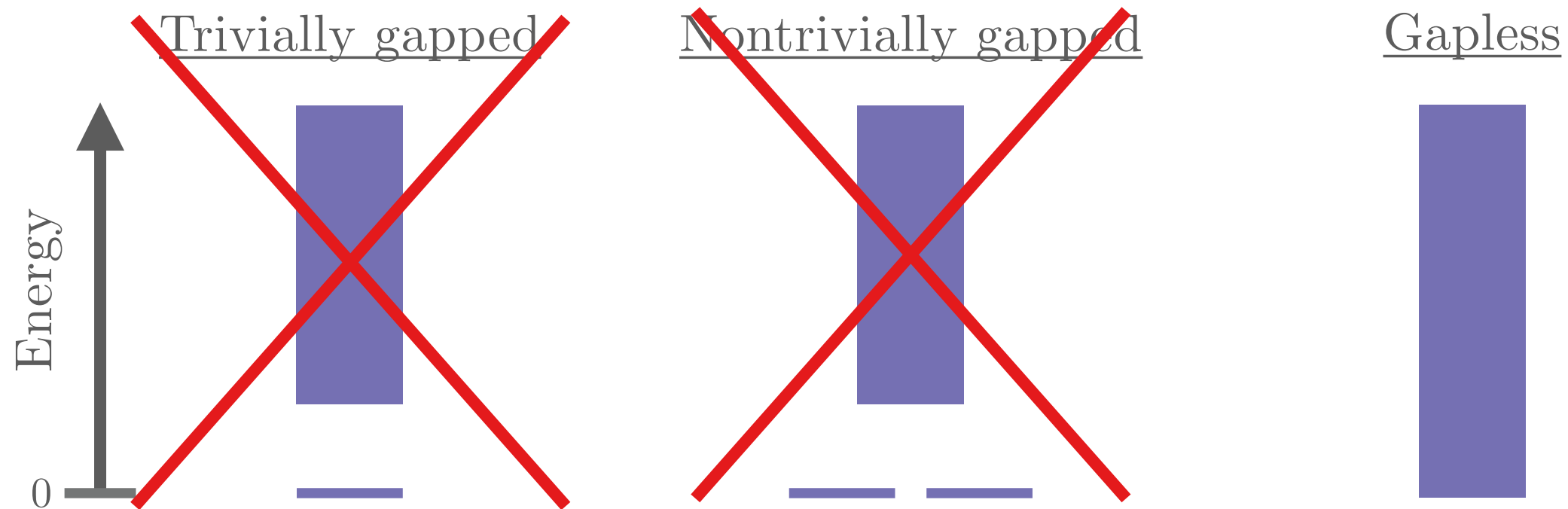
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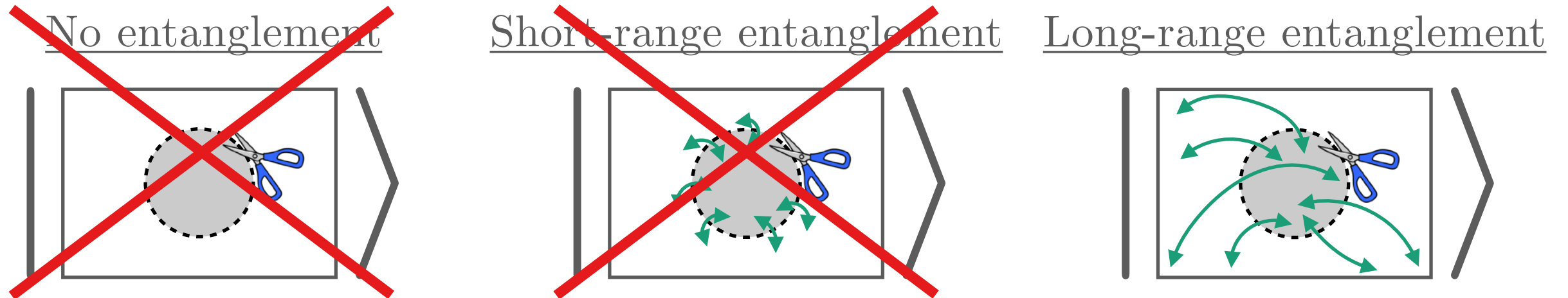
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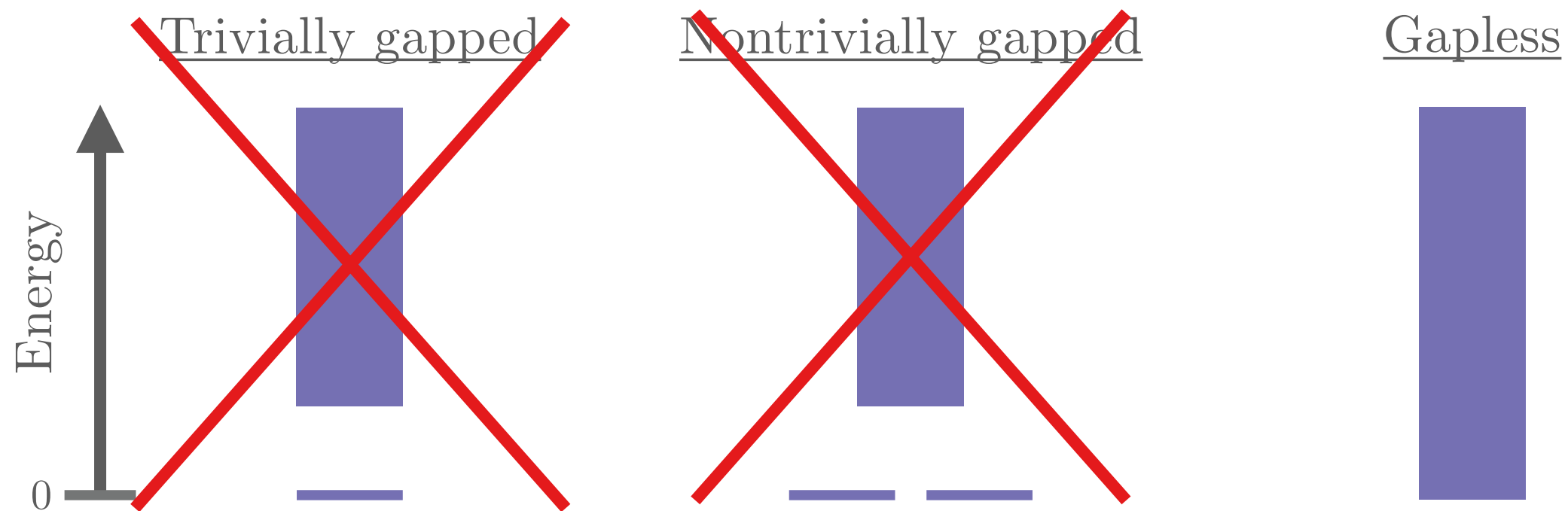


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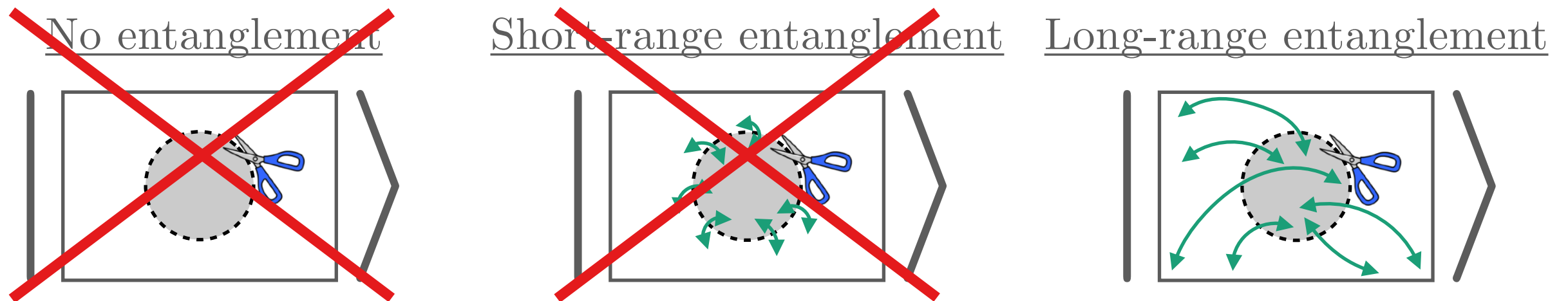


True for ordinary and
generalized symmetries!

1. Constrains low-energy dynamics



2. Constrains ground state entanglement



Conclusion

Symmetry is both a foundation and frontier of physics

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➤ Amazingly interdisciplinary frontier!

Condensed Matter

Generalized Landau paradigm, New phases of matter, New LSM theorems, Anyons & fractons, Quantum disordering

High Energy

Confinement, Anomaly matching, Phenomenology, Geometric engineering, Amplitudes, Quantum gravity

Symmetry

Quantum Information

Sequential circuits, Fault-tolerant gates, Quantum cellular automata, Matrix product operators, ZX-calculus

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Category theory, Hopf algebra, Operator algebra, Cobordism, Stable homotopy, Homological algebra

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